Basic Algebra Rules

1. Fractions. Let $a, b, c$, and $d$ be numbers.

   (a) You can break up a fraction from a sum in the numerator, but *not* in the denominator:
   
   $$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$
   
   but
   
   $$\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$$

   (b) Cancellation of the $c$ here requires that it appears in *each* additive term of the numerator and denominator:
   
   $$\frac{ca+cb}{cd} = \frac{c(a+b)}{cd} = \frac{a+b}{d}$$
   
   but
   
   $$\frac{ca+b}{cd} \neq \frac{a+b}{d}$$

   (c) Compound fractions can be simplified by using the rule “division is the same as multiplication by the reciprocal”:
   
   $$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

2. Natural Logs. Let $a$ and $b$ be numbers.

   (a) Natural logs distribute in a funny way over products and quotients:
   
   $$\ln (ab) = \ln a + \ln b$$

   $$\ln \left(\frac{a}{b}\right) = \ln a - \ln b$$

   but they do *not* distribute over sums:

   $$\ln (a+b) \neq \ln a + \ln b$$

   (b) Natural logs can help you work with exponents by ”brining them down”:

   $$\ln (a^b) = b \ln a$$
3. Exponents. Let \(a, b, m,\) and \(n\) be numbers.

(a) Exponents distribute over products, but not over sums:

\[
(ab)^n = a^n b^n
\]

but

\[
(a + b)^n \neq a^n + b^n
\]

(b) A negative exponent can always be viewed as a denominator, and vice versa:

\[
a^{-n} = \frac{1}{a^n}
\]

(c) Two terms with exponents can only be multiplied if they share the same base; in that case, the exponents add:

\[
a^m a^n = a^{m+n}
\]

but \(a^m d^n\) cannot be further simplified, and

\[
a^m a^n \neq a^{mn}
\]

(d) Similarly for division:

\[
\frac{a^m}{a^n} = a^{m-n}
\]

4. Roots. Let \(a, b, m,\) and \(n\) be numbers.

(a) Remember that roots can always be viewed as fractional exponents:

\[
\sqrt[n]{a} = a^{\frac{1}{n}}
\]

With this point of view, we’ll inherit all the rules about exponents. In particular,

(b) Distributing a root over a product:

\[
\sqrt[n]{ab} = (ab)^{\frac{1}{n}} = a^{\frac{1}{n}} b^{\frac{1}{n}} = \sqrt[n]{a} \sqrt[n]{b}
\]

(c) Multiplying two roots with a common base:

\[
\sqrt[n]{a} \sqrt[n]{a} = a^{\frac{1}{n}} a^{\frac{1}{n}} = a^{\frac{1}{n} + \frac{1}{n}}
\]