Name: ______________________ **PID:** ______________

TA: __________________ **Sec. No:** ____ **Sec. Time:** ____

Math 20D.
Final Examination
December 9, 2008

Turn off and put away your cell phone.
No calculators or any other electronic devices are allowed during this exam.
You may use one page of notes, but no books or other assistance during this exam.
Read each question carefully, and answer each question completely.
Show all of your work; no credit will be given for unsupported answers.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

#	Points	Score
1	10	
2	10	
3	10	
4	8	
5	8	
6	10	
Σ	56	

1. Consider the following differential equation:

$$(x-2)y'' + y = 0.$$

(a) (3 points) Find the recurrence relation for the power series solution about the point $x_0 = 0$.

(b) (4 points) Write the first four terms of each of the two linearly independent power series solutions.

(c) (3 points) What is a lower bound for the radius of convergence of a power series solution about $x_0 = 0$? Be sure to explain how you arrived at your answer.

2. (10 points) Use the Laplace transform to solve the following initial value problem:

$$\begin{cases} y'' - y = \delta(t-2) + u_2(t) \\ y(0) = 2, \quad y'(0) = 2 \end{cases}$$

Refer to the attached table of Laplace transforms.

3. Consider the following homogeneous linear system of differential equations:

$$\mathbf{x}' = \begin{pmatrix} 2 & -4 \\ 5 & -2 \end{pmatrix} \mathbf{x}.$$

(a) (4 points) The eigenvalues of the coefficient matrix are complex. Find the eigenvalues and for each eigenvalue, find an associated eigenvector.

(b) (2 points) Find two linearly independent complex-valued solutions to the system of differential equations.

(c) (4 points) Find two linearly independent *real-valued* solutions to the linear system of differential equations. (Continue your solution on the next page, if necessary.)

4. Consider the differential equation

$$x^3 - 3xy^2 + (y^3 - 3x^2y)y' = 0.$$

(a) (2 points) Verify that the differential equation is exact.

(b) (6 points) Solve the differential equation. Leave the result in implicit form: do not try to solve for $y(x)$ explicitly.

5. (8 points) Solve the linear first-order initial value problem

$$\begin{cases} ty' + 4y = \frac{\sin(t)}{t^3} \\ y\left(\frac{\pi}{2}\right) = 4 \end{cases}$$

6. Consider the following nonhomogeneous linear system of differential equations:

$$\mathbf{x}' = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-3t}.$$

(a) (4 points) Find the eigenvalues of the coefficient matrix and for each eigenvalue, find an associated eigenvector.

(b) (2 points) Write the general solution to the corresponding homogeneous system.

(c) (4 points) Use undetermined coefficients to find the general solution to the nonhomogeneous system. (Continue your solution on the next page, if necessary.)

Table of Elementary Laplace Transforms

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\frac{1}{s}, \quad s > 0$
2.	e^{at}	$\frac{1}{s-a}, \quad s > a$
3.	$t^n, \quad n = \text{positive integer}$	$\frac{n!}{s^{n+1}}, \quad s > 0$
4.	$t^p, \quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5.	$\sin(at)$	$\frac{a}{s^2+a^2}, \quad s > 0$
6.	$\cos(at)$	$\frac{s}{s^2+a^2}, \quad s > 0$
7.	$\sinh(at)$	$\frac{a}{s^2-a^2}, \quad s > \lvert a \rvert$
8.	$\cosh(at)$	$\frac{s}{s^2-a^2}, \quad s > \lvert a \rvert$
9.	$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$
10.	$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$
11.	$t^n e^{at}, \quad n = \text{positive integer}$	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
12.	$u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$
13.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
16.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$
19.	$(-t)^n f(t)$	$F^{(n)}(s)$