Math 20E

September 2, 2009

Question 1 Suppose $\mathbf{F}$ is a conservative vector field on $\mathbb{R}^3$. Then,

A. $\boldsymbol{\nabla} \cdot \mathbf{F} = k$ for some constant k.

B. $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{s} = 0$ along every oriented simple closed curve C.

C. $\boldsymbol{\nabla} \times \mathbf{F} = \mathbf{0}$

D. There is a scalar function $f : \mathbb{R}^3 \to \mathbb{R}$ for which $\mathbf{F} = \boldsymbol{\nabla} f$

***E.** **B**, **C** and **D**

Question 2 Suppose $\mathbf{F}$ is a C^1 vector field for which $\nabla \cdot \mathbf{F} = 0$. Then,

A. If $\mathbf{F}$ is the velocity field of a fluid, the fluid is said to be incompressible.

B. $\nabla \times \mathbf{F} = \mathbf{0}$

C. There is a vector field $\mathbf{G}$ such that $\nabla \times \mathbf{G} = \mathbf{F}$

D. **A**, **B** and **C**

*__E.__ **A** and **C** and the vector field $\mathbf{G}$ in part **C** is called a vector potential for $\mathbf{F}$

Question 3 Suppose $\mathbf{F}$ is a C^1 vector field on the unit sphere S in $\mathbb{R}^3$. Then, $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$

A. is 0

B. is most easily computed by parametrizing S using spherical coordinates.

C. is most easily computed by applying Stokes' theorem and computing $\int_{\partial S} \mathbf{F} \cdot d\mathbf{s}$

D. cannot be computed using Stokes' theorem because the sphere S has no boundary curve ∂S

*__E.__ **A** and **C**: the line integral in **C** is 0 because the boundary curve ∂S is empty