

Math 20E

August 26, 2009

Question 1 Given a C^1 surface with two distinct parameterizations $\Phi : D \rightarrow S$ and $\Psi : D \rightarrow S$, then

A. $\iint_{\Phi} \mathbf{F} \cdot d\mathbf{S} = \iint_{\Psi} \mathbf{F} \cdot d\mathbf{S}$

B. $\iint_{\Phi} \mathbf{F} \cdot d\mathbf{S} = \iint_{\Psi} \mathbf{F} \cdot d\mathbf{S}$ when $\frac{\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v}}{\|\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v}\|} = \frac{\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}}{\|\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}\|}$

C. $\iint_{\Phi} \mathbf{F} \cdot d\mathbf{S} = - \iint_{\Psi} \mathbf{F} \cdot d\mathbf{S}$ when $\frac{\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v}}{\|\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v}\|} = - \frac{\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}}{\|\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}\|}$

D. $\iint_{\Phi} \mathbf{F} \cdot d\mathbf{S} < \iint_{\Psi} \mathbf{F} \cdot d\mathbf{S}$ when $\|\frac{\partial \Phi}{\partial u} \times \frac{\partial \Phi}{\partial v}\| < \|\frac{\partial \Psi}{\partial u} \times \frac{\partial \Psi}{\partial v}\|$

***E. B and C**

Question 2 Given a C^1 surface S parameterized by $\Phi : D \rightarrow S$, where

$$\Phi(u, v) = (x(u, v), y(u, v), z(u, v)),$$

and given $\mathbf{F}(x, y, z)$ a continuous vector field defined on S . Then,

A. $\mathbf{n} = \frac{\mathbf{T}_u \times \mathbf{T}_v}{\|\mathbf{T}_u \times \mathbf{T}_v\|}$ is the positively oriented unit normal vector to S specified by Φ .

B.
$$\iint_S \mathbf{F} \cdot \mathbf{n} \, dS = \iint_S \mathbf{F} \cdot d\mathbf{S}$$

C. The average value of $\mathbf{F} \cdot \mathbf{n}$ on S is $\frac{1}{A(S)} \iint_S \mathbf{F} \cdot d\mathbf{S}$,
where $A(S) = \iint_D \|\mathbf{T}_u \times \mathbf{T}_v\| \, du \, dv$

D. **A** and **B**

***E.** **A**, **B** and **C**