Name: ______________________ **PID:** ______________

TA: ____________________ **Sec. No:** _____ **Sec. Time:** _____

Math 20E.
Final Examination
September 6, 2008

Turn off and put away your cell phone.
Any type of calculator is allowed, but no laptops or other devices are allowed on this exam.
You may use one page of notes, but no books or other assistance on this exam.
Read each question carefully, answer each question completely, and show all of your work.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

#	Points	Score
1	6	
2	6	
3	6	
4	8	
5	6	
6	6	
7	6	
8	6	
Σ	50	

1. (6 points) Evaluate

$$\int_{y=0}^{1} \int_{x=\sqrt{y}}^{1} \sin\left(x^3\right) \, dx \, dy.$$

by changing the order of integration. (Note: you need not simplify trigonometric expressions in your final answer.)

2. (6 points) Find an appropriate change of variables to evaluate

$$\iint_R (x+y)\, e^{x^2-y^2}\, dx\, dy,$$

where R is the square with vertices $(2,0)$, $(0,2)$, $(-2,0)$, $(0,-2)$. (Hint: first determine equations for each side of the square R.)

3. Consider the vector field

$$\mathbf{F}(x, y, z) = \big(x - yz\sin(xyz),\ y - xz\sin(xyz),\ -xy\sin(xyz)\big).$$

(a) (3 points) Find a scalar function $f(x, y, z)$ such that $\mathbf{F} = \boldsymbol{\nabla} f$.

(b) (3 points) Evaluate the line integral $\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s}$ along the line segment connecting $(0, 0, 0)$ to $(1, 1, 3\pi)$.

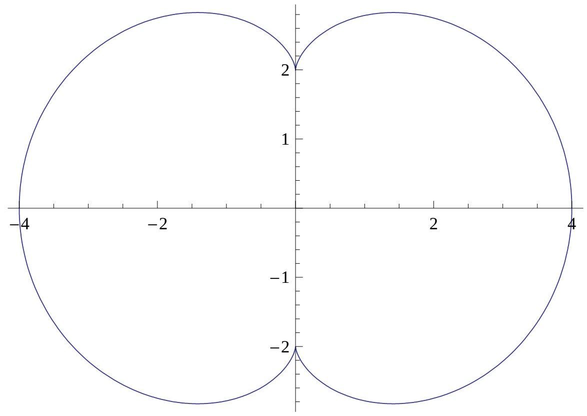
4. Let S be the paraboloid $z = 4 - x^2 - y^2$ with $z \geq 0$.

(a) (4 points) Determine a parameterization for S. Be sure to clearly state the domain of your parameterization.

(b) (4 points) Find the area of S.

5. (6 points) Use Green's theorem to find the area enclosed by the epicycloid

$$\mathbf{c}(t) = (3\sin(t) - \sin(3t),\ 3\cos(t) - \cos(3t)), \quad 0 \le t \le 2\pi.$$



6. (6 points) Let $\mathbf{F}(x, y, z) = (z - 2y,\ x - z,\ 2y - x)$. Use Stokes' theorem to compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{s}$ along the circle C of radius 2 centered at $(0, 0, 0)$ and lying in the plane $3x + 2y + 3z = 0$.

7. (6 points) Let $\mathbf{F}(x, y, z) = (2x - y,\ -2y + z,\ 2y - x)$. Use the divergence theorem to compute the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where S is the unit sphere $x^2 + y^2 + z^2 = 1$.

8. (6 points) Use Stokes' theorem to evaluate $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$, where S is the portion of the surface $z = 4 - x^2 - y^2$ with $z \geq 0$, and $\mathbf{F}(x, y, z) = (2y,\ -2x,\ ze^{xy})$.

Math 20E
Formula Sheet

Vector Identities

1. $\nabla(f+g) = \nabla f + \nabla g$
2. $\nabla(cf) = c\,\nabla f$
3. $\nabla(fg) = f\,\nabla g + g\,\nabla f$
4. $\nabla\left(\frac{f}{g}\right) = \frac{g\,\nabla f - f\,\nabla g}{g^2}$, at points $\mathbf{x}$ where $g(\mathbf{x}) \neq 0$
5. $\nabla\cdot(\mathbf{F}+\mathbf{G}) = \nabla\cdot\mathbf{F} + \nabla\cdot\mathbf{G}$
6. $\nabla\times(\mathbf{F}+\mathbf{G}) = \nabla\times\mathbf{F} + \nabla\times\mathbf{G}$
7. $\nabla\cdot(f\mathbf{F}) = f\,\nabla\cdot\mathbf{F} + \mathbf{F}\cdot\nabla f$
8. $\nabla\times(f\mathbf{F}) = f\,\nabla\times\mathbf{F} + \nabla f\times\mathbf{F}$
9. $\nabla\cdot(\nabla\times\mathbf{F}) = 0$
10. $\nabla\times\nabla f = \mathbf{0}$
11. $\nabla\cdot(\mathbf{F}\times\mathbf{G}) = \mathbf{G}\cdot(\nabla\times\mathbf{F}) - \mathbf{F}\cdot(\nabla\times\mathbf{G})$

Selected Integrals

1. $\int \tan(x)\,dx = \log|\sec(x)|$
2. $\int \sec(x)\,dx = \log|\sec(x)+\tan(x)|$
3. $\int \arcsin\left(\frac{x}{a}\right)\,dx = x\arcsin\left(\frac{x}{a}\right) + \sqrt{x^2+a^2}$
4. $\int \arctan\left(\frac{x}{a}\right)\,dx = x\arctan\left(\frac{x}{a}\right) - \frac{a}{2}\log(a^2+x^2)$
5. $\int \sin^2(x)\,dx = \frac{1}{2}\left(x - \sin(x)\cos(x)\right)$
6. $\int \cos^2(x)\,dx = \frac{1}{2}\left(x + \sin(x)\cos(x)\right)$
7. $\int \frac{1}{\sqrt{a^2+x^2}}\,dx = \log(x+\sqrt{a^2+x^2})$
8. $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right)$
9. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\arcsin\left(\frac{x}{a}\right)$
10. $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\left(\frac{x}{a}\right)$
11. $\int \frac{1}{\sqrt{x^2\pm a^2}}\,dx = \log(x+\sqrt{x^2\pm a^2})$
12. $\int \frac{1}{(a^2-x^2)^{3/2}}\,dx = \frac{x}{a^2\sqrt{a^2-x^2}}$
13. $\int \sqrt{x^2\pm a^2}\,dx = \frac{x}{2}\sqrt{x^2\pm a^2} \pm \frac{a^2}{2}\log(x+\sqrt{x^2\pm a^2})$
14. $\int (a^2-x^2)^{3/2}\,dx = \frac{x}{8}(5a^2-2x^2)\sqrt{a^2-x^2} + \frac{3a^4}{8}\arcsin\left(\frac{x}{a}\right)$

Path and Line Integrals over a Parameterized Curve (Path) $\mathbf{c}(t)$

1. Path integral of a scalar function f: $\displaystyle\int_{\mathbf{c}} f\,ds = \int_a^b f(\mathbf{c}(t))\,\|\mathbf{c}'(t)\|\,dt$
2. Line integral of a vector field $\mathbf{F}$: $\displaystyle\int_{\mathbf{c}} \mathbf{F}\cdot d\mathbf{s} = \int_a^b \mathbf{F}(\mathbf{c}(t))\cdot\mathbf{c}'(t)\,dt$

Surface Integrals over a Parameterized Surface $\mathbf{\Phi}(u,v)$

1. Integral of a scalar function f: $\displaystyle\iint_S f\,dS = \iint_D f(\mathbf{\Phi}(u,v))\,\|\mathbf{T}_u\times\mathbf{T}_v\|\,du\,dv$
2. Integral of a vector field $\mathbf{F}$: $\displaystyle\iint_S \mathbf{F}\cdot d\mathbf{S} = \iint_D \mathbf{F}(\mathbf{\Phi}(u,v))\cdot(\mathbf{T}_u\times\mathbf{T}_v)\,du\,dv$

Green's Theorem: $\displaystyle\int_{\partial D} P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dx\,dy$

Special Case: $\displaystyle\frac{1}{2}\int_{\partial D} x\,dy - y\,dx = \iint_D dx\,dy$

Stoke's Theorem: $\displaystyle\iint_S (\boldsymbol{\nabla}\times\mathbf{F})\cdot d\mathbf{S} = \int_{\partial S}\mathbf{F}\cdot d\mathbf{s}$

Gauss' Theorem: $\displaystyle\iiint_W (\boldsymbol{\nabla}\cdot\mathbf{F})\,dV = \iint_{\partial W}\mathbf{F}\cdot d\mathbf{S}$