Math 142B
Midterm Exam 2
May 18, 2012

Instructions
1. You may use any type of calculator, but no other electronic devices during this exam.
2. You may use one page of notes, but no books or other assistance during this exam.
3. Write your Name, PID, and Section on the front of your Blue Book.
4. Write your solutions clearly in your Blue Book
   (a) Carefully indicate the number and letter of each question and question part.
   (b) Present your answers in the same order they appear in the exam.
   (c) Start each question on a new side of a page.
5. Read each question carefully, and answer each question completely.
6. Show all of your work; no credit will be given for unsupported answers.

1. Given \( f : [a, b] \to \mathbb{R} \) continuous. Use the Second Fundamental Theorem and additivity of the integral to show that
\[
\frac{d}{dx} \left[ \int_x^b f \right] = -f(x) \quad \text{for all } x \in (a, b).
\]
(Note: The definition that \( \int_a^b f = -\int_b^a f \) is based on this result. Thus, your proof should not appeal to this definition.)

2. Given a polynomial \( p \) of degree at most \( n \) and \( x_0 \) any point. Show that the \( n \)th Taylor polynomial for \( p \) at \( x_0 \) is \( p \) itself. You may assume that \( p \) can be written in the form \( p(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)^2 + \cdots + a_n(x - x_0)^n \).

3. Given the function \( f : [0, 1] \to \mathbb{R} \) defined by
\[
f(x) = \begin{cases} 
1 & \text{if } 0 \leq x \leq \frac{1}{2}, \\
2 & \text{if } \frac{1}{2} < x \leq 1.
\end{cases}
\]
As we’ve seen, \( f \) is integrable and \( \int_0^1 f = \frac{3}{2} \). Show that the Mean Value Theorem for Integrals does not hold for \( f \) and explain why \( f \) does not satisfy the hypotheses for the Mean Value Theorem for Integrals.

4. Given a number \( r \) with \( 0 < r < 1 \), let \( f : [-r, r] \to \mathbb{R} \) be defined by \( f(x) = (1 - x)^{-1} \).
   (a) Find a formula for the \( n \)th Taylor polynomial for \( f \) at 0.
   (b) Use the Lagrange Remainder Theorem to show that the Taylor series for \( f \) at 0 converges to \( f(x) \) for all \( x \in [-r, r] \).