Question 1  The surface $x^2 + y^2 + z = 1$ for $z \geq 0$ is parametrized by $\Phi : D \rightarrow \mathbb{R}^3$, where $D$ is the unit disk $u^2 + v^2 \leq 1$ and $\Phi(u, v) = (u, v, 1 - u^2 - v^2)$. Then, $T_u \times T_v = (2u, 2v, 1)$ and

A. $\Phi$ is a one-to-one mapping of $D$ onto $S = \Phi(D)$.

B. The parametrized surface $\Phi$ is regular at every point of $S$.

C. The surface $S$ has a tangent plane at every point.

*D. A, B and C

E. none of the above
Question 2  The surface $x^2 + y^2 + z = 1$ for $z \geq 0$ is parametrized by $\Psi : R \to \mathbb{R}^3$, where $R$ is the rectangle $[0,1] \times [0, 2\pi]$ and $\Psi(u,v) = (u \cos(v), u \sin(v), 1 - u^2)$. Then, $T_u \times T_v = u(2u \cos(v), 2u \sin(v), u)$ and

A. $\Psi$ is a one-to-one mapping of $R$ onto $S = \Psi(R)$.

B. The parametrized surface $\Psi$ is regular at every point of $S$.

*C. The surface $S$ has a tangent plane at every point.

D. A, B and C

E. none of the above
Question 3  The surface $S$ given by $x^2 + y^2 + z = 1$ for $z \geq 0$ is parametrized by $\Phi : D \to \mathbb{R}^3$, where $D$ is the unit disk $u^2 + v^2 \leq 1$ and $\Phi(u, v) = (u, v, 1 - u^2 - v^2)$. $S$ is also parametrized by $\Psi : R \to \mathbb{R}^3$, where $R$ is the rectangle $[0, 1] \times [0, 2\pi]$ and $\Psi(r, \theta) = (r \cos(\theta), r \sin(\theta), 1 - r^2)$.

A. $\Phi$ is a one-to-one mapping of $D$ onto $S = \Phi(D)$.

B. $\Psi$ is a one-to-one mapping of $R$ onto $S = \Psi(R)$.

C. $\Psi = \Phi \circ T$, where $T : R \to D$ is the polar coordinate transformation.

D. A, B and C

*E. A and C