Math 109

August 5, 2016

Question 1 The easiest way to prove an implication of the form (P or Q) $\Rightarrow R$ is to prove

A. ($P \Rightarrow R$) and ($Q \Rightarrow R$).

B. its contrapositive (not R) $\Rightarrow$ (not P) and (not Q).

C. [(not R) $\Rightarrow$ (not P)] and [(not R) $\Rightarrow$ (not Q)].

D. **B** or **C**, since they're logically equivalent.

***E.** **A**, **B**, or **C**, since they're all logically equivalent.

Question 2 De Morgan's Law is the logical relationship

A. not $(P$ or $Q) \Leftrightarrow$ (not P) and (not Q).

B. not $(P$ and $Q) \Leftrightarrow$ (not P) or (not Q).

***C.** **A** and **B**: both are part of De Morgan's Law(s).

D. Neither **A** nor **B**: both equivalencies are false.

E. There's no De Morgan's Law! Who was De Morgan anyway?

Question 3 The implication $P \Rightarrow (Q \text{ or } R)$ can be proved by

A. proving $[P \text{ and } (\text{not } Q)] \Rightarrow R$.

B. proving $[P \text{ and } (\text{not } R)] \Rightarrow Q$.

C. cases: that is, by proving $(P \Rightarrow R)$ and $(Q \Rightarrow R)$.

*__D.__ **A** or **B** since they are logically equivalent.

E. None of the above: $P \Rightarrow (Q \text{ or } R)$ must be proved directly.

Question 4 **Theorem:** All horses are the same color.

Proof: Clearly, the theorem holds when there is $n = 1$ horse. Suppose the theorem holds for n horses. Given a set of $n + 1$ horses, the inductive hypothesis implies that the 1$^{\text{st}}$ through n^{th} and 2$^{\text{nd}}$ through $(n+1)^{\text{st}}$ horses are all the same color. Hence, all $n + 1$ horses are the same color and the theorem follows.

A. The theorem is balderdash, since it is easy to exhibit two horses with different colors.

B. The proof is faulty because mathematical induction cannot be applied to horses.

C. The proof is faulty because because it fails at the inductive step from $n = 1$ to $n = 2$.

D. **A** and **B**

*__E.__ **A** and **C**

Question 5 **Theorem:** $n^2 \leq n$ for every positive integer n.

Proof: Clearly, $1^2 \leq 1$. Suppose $k^2 \leq k$ for some positive integer k. If $(k+1)^2 \leq (k+1)$, then $k^2 \leq (k+1)^2 - 1 \leq (k+1) - 1 = k$. Thus, $(k+1)^2 \leq k+1$ must be true.

A. The theorem is balderdash. For example, $2^2 > 2$.

B. Since the proof is clear, this has become known as the Square Paradox.

C. The proof is faulty because it assumes the conclusion as part of the argument.

D. **A** and **B**

*__E.__ **A** and **C**