

Math 20E

August 8, 2016

Question 1 Consider the iterated integral

$$\int_{y=0}^{\sqrt{\pi/2}} \int_{x=y}^{\sqrt{\pi/2}} \sin(x^2) dx dy.$$

Which best describes how it might be evaluated?

- A.** The iterated integral can easily be integrated by evaluating it as written, integrating with respect to x first and then y .
- B.** The iterated integral is impossible to integrate analytically because the antiderivative of $\sin(x^2)$ cannot be expressed in terms of elementary functions.
- C.** The iterated integral should be expressed as

$$\iint_D \sin(x^2) dA$$

for an appropriate region D .

- D.** The iterated integral should be rewritten as an iterated integral with respect to y first and then x .
- *E.** Both **C** and **D**: one should determine the region D in **C** in order to determine the appropriate limits of integration for the re-ordered iterated integral in **D**.

Question 2 Consider the triple integral

$$\int_{z=p}^q \int_{y=c}^d \int_{x=a}^b f(x, y, z) dx dy dz.$$

Which of the following best describes the possibility(s) for the order of integration?

- A.** There is only one possible order of integration: the one given.
- B.** There are two possible orders of integration: x and y may be switched, as in double integrals.
- C.** There are three possible orders of integration: one for each variable.
- *D.** There are six possible orders of integration: one for each permutation of (x, y, z) .
- E.** None of the above: not enough information is given about the region of integration to decide.

Question 3 Given an elementary region D in \mathbb{R}^2 and a function $f : D \rightarrow \mathbb{R}$ for which the double integral $\iint_D f(x, y) dA$ exists. The number $\bar{f}_D = \frac{1}{D} \iint_D f(x, y) dA$ is called the mean (or, average) value of the function f on D . Then:

- A.** If m_D and M_D are the maximum and minimum values of f on D , respectively, then $m_D \leq \bar{f}_D \leq M_D$. This is called the mean value inequality.
- B.** There is a point $(x_0, y_0) \in D$ at which $f(x_0, y_0) = \bar{f}_D$. This is called the mean value theorem.
- C.** There is a point $(x_0, y_0) \in D$ at which $f(x_0, y_0) = \bar{f}_D$, provided f is continuous on D . This is called the mean value theorem.
- D.** **A** and **B**
- *E.** **A** and **C**

Question 4 Recall that two vectors $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$ are orthogonal if and only if $\mathbf{v} \cdot \mathbf{w} = 0$. Which of the following statements are true?

- A.** If \mathbf{v} and \mathbf{w} are orthogonal, then the angle between \mathbf{v} and \mathbf{w} is $\frac{\pi}{2}$.
- B.** If $\mathbf{c} : [a, b] \rightarrow \mathbb{R}^n$ is a C^1 path, then $\mathbf{c}'(t)$ is a tangent vector to $\mathbf{c}(t)$ at every $t \in [a, b]$.
- C.** $\mathbf{0}$ is the only vector in \mathbb{R}^n that is orthogonal to itself.
- D.** **B** is true when \mathbf{c} is a regular path.
- *E.** **C** and **D**