Math 20E

August 15, 2016

**Question 1** Given a $C^1$ path $\mathbf{c} : [a, b] \to \mathbb{R}^n$. $\mathbf{c}$ is *regular* at $t_0$ means

**A.** $\mathbf{c}'(t_0) = \mathbf{0}$.

*__B.__ $\mathbf{c}'(t_0) \neq \mathbf{0}$.

**C.** the image curve $\mathbf{c}([a, b])$ has a tangent line at $\mathbf{c}(t_0)$.

**D.** $\mathbf{c}'(t_0)$ is a unit vector.

**E.** both **B** and **C**.

**Question 2** Two paths $\mathbf{c}_1 : [0, 2\pi] \to \mathbb{R}^3$ and $\mathbf{c}_2 : [0, 2\pi] \to \mathbb{R}^3$ are given by

$$\mathbf{c}_1(t) = \big(\cos(t), \sin(t), t\big)$$
$$\mathbf{c}_2(t) = \big(\cos(2\pi - t), \sin(2\pi - t), 2\pi - t\big)$$

The lengths of the corresponding curves are

*__A.__ the same since the curves defined by the paths are the same.

**B.** of opposite sign since the paths traverse the curves in opposite directions.

**C.** cannot be computed because the antiderivative of $||\mathbf{c}_1'(t)||$ and $||\mathbf{c}_2'(t)||$ cannot be computed.

**D.** both **B** and **C**

**E.** none of the above

**Question 3** Two paths $\mathbf{c}_1 : [0, 2\pi] \to \mathbb{R}^3$ and $\mathbf{c}_2 : [0, 2\pi] \to \mathbb{R}^3$ are given by

$$\mathbf{c}_1(t) = \big(\cos(t), \sin(t), t\big)$$
$$\mathbf{c}_2(t) = \big(\cos(2\pi - t), \sin(2\pi - t), 2\pi - t\big)$$

Let $\mathbf{F}(x, y, z)$ be any $C^1$ vector field. The value of the line integrals $\int_{\mathbf{c}_1} \mathbf{F} \cdot d\mathbf{s}$ and $\int_{\mathbf{c}_2} \mathbf{F} \cdot d\mathbf{s}$ are

**A.** the same since the curves defined by the paths are the same.

*__B.__ of opposite sign since the paths traverse the curves in opposite directions.

**C.** cannot be computed because the antiderivative of $||\mathbf{c}_1'(t)||$ and $||\mathbf{c}_2'(t)||$ cannot be computed.

**D.** both **B** and **C**

**E.** none of the above

**Question 4** Given $f : \mathbb{R}^n \to \mathbb{R}$ a $C^1$ scalar-valued function and $\mathbf{c} : [a, b] \to \mathbb{R}^n$ a simple $C^1$ path,

**A.** $\mathrm{D}f(t) = \nabla f(\mathbf{c}(t)) \cdot \mathbf{c}'(t)$ by the chain rule.

**B.** $\displaystyle\int_{\mathbf{c}} \nabla f \cdot d\mathbf{s} = \int_a^b \nabla f(\mathbf{c}(t)) \cdot \mathbf{c}'(t)\, dt.$

**C.** The value of $\displaystyle\int_{\mathbf{c}} \nabla f \cdot d\mathbf{s}$ is independent of the path $\mathbf{c}$.

**D.** $\displaystyle\int_{\mathbf{c}} \nabla f \cdot d\mathbf{s} = f(\mathbf{c}(b)) - f(\mathbf{c}(a))$, which depends only on the value of $f$ at the endpoints $\mathbf{c}(a)$ and $\mathbf{c}(b)$.

*__E.__ All of the above.