Math 20E

August 29, 2016

Question 1 Given a one-to-one parametrized surface $\Phi : D \to S \subset \mathbb{R}^3$. Then,

A. $\Phi : \partial D \to S$ parametrizes the boundary curve $\partial\Phi$ of the parametrized surface Φ.

B. The orientation of $\partial\Phi$ is determined by the orientation of ∂D.

C. Stokes' Theorem on Φ follows from Green's Theorem on D.

D. All of the above.

*__E.__ None of the above. George Stokes and George Green never met.

Question 2 Suppose $\mathbf{F}$ is a C^1 vector field on the unit sphere S in $\mathbb{R}^3$. Then, $\displaystyle\iint_S (\boldsymbol{\nabla}\times\mathbf{F})\cdot d\mathbf{S}$

A. is 0.

B. is most easily computed by parametrizing S using latitude-longitude coordinates.

C. is most easily computed by applying Stokes' theorem and computing $\displaystyle\int_{\partial S} \mathbf{F}\cdot d\mathbf{s}$.

D. cannot be computed using Stokes' theorem because the sphere S has no boundary curve ∂S.

*__E.__ **A** and **C**: the line integral in **C** is 0 because the boundary curve ∂S is empty.

Question 3 Suppose $\mathbf{F}$ is a C^1 vector field on $\mathbb{R}^3$. Let H be the unit hemisphere given by $x^2 + y^2 + z^2 = 1$ with $z \geq 0$, let D be the unit disk given by $z = 0$ with $x^2 + y^2 \leq 1$, and let $S = H \cup D$. Then,

A. $\partial H = \partial D$, including orientation when H and D are both oriented with the upward-pointing unit normal vector.

B. $\displaystyle\iint_H (\boldsymbol{\nabla} \times \mathbf{F}) \cdot d\mathbf{S} = \iint_D (\boldsymbol{\nabla} \times \mathbf{F}) \cdot d\mathbf{S}.$

C. $\displaystyle\iint_S (\boldsymbol{\nabla} \times \mathbf{F}) \cdot d\mathbf{S} = \iint_H (\boldsymbol{\nabla} \times \mathbf{F}) \cdot d\mathbf{S} + \iint_D (\boldsymbol{\nabla} \times \mathbf{F}) \cdot d\mathbf{S}.$

D. $\displaystyle\iint_S (\boldsymbol{\nabla} \times \mathbf{F}) \cdot d\mathbf{S} = 0.$

*__E.__ **A**, **B** and **D**