Math 142A Homework Assignment 5
Due Wednesday, November 15

1. Show that $f : [1, \infty) \to \mathbb{R}$ given by $f(x) = \sqrt{x}$ satisfies the $\varepsilon - \delta$ criterion on $[1, \infty)$. Conclude that f is uniformly continuous.

2. Show that if $f : (a, b) \to \mathbb{R}$ is uniformly continuous, then f is bounded; that is, $f\left((a, b)\right)$ is bounded.

3. Exhibit an example of

   (a) a continuous function $f : (0, 1) \to \mathbb{R}$ that is not bounded.

   (b) a bounded continuous function $g : (0, 1) \to \mathbb{R}$ that is not uniformly continuous.

4. Show that any function $f : \mathbb{Z} \to \mathbb{R}$ is uniformly continuous.

   [Recall that $\mathbb{Z}$ is the set of integers.]

5. Let $f : [0, \infty) \to \mathbb{R}$ be a monotone function. Prove that $\lim_{x\to\infty} f(x) = L$ for some number L if and only if $f\left([0, \infty)\right)$ is bounded.

6. $f : \mathbb{R} \to \mathbb{R}$ is said to be *odd* if $f(-x) = -f(x)$ for all x. Show that if $f : \mathbb{R} \to \mathbb{R}$ is odd and $f|_{[0,\infty)}$ is strictly increasing, then f is strictly increasing.

   [Note: $f|_{[0,\infty)}$ means $f : [0, \infty) \to \mathbb{R}$, the restriction of f to $[0, \infty)$.]

7. Show that if $f : [a, b] \to \mathbb{R}$ is a monotone function satisfying the intermediate value property, then f is continuous.