

Name: _____ PID: _____

TA: _____ Sec. No: _____ Sec. Time: _____

Math 142A
Midterm Exam 1
January 28, 2011

Turn off and put away your cell phone.

No calculators or any other electronic devices are allowed during this exam.

You may use one page of notes, but no books or other assistance on this exam.

Read each question carefully, answer each question completely, and show all of your work.

Write your solutions clearly and legibly; no credit will be given for illegible solutions.

If any question is not clear, ask for clarification.

#	Points	Score
1	6	
2	6	
3	6	
4	6	
Σ	24	

1. Let $S = \{\frac{1}{n} \mid n \text{ in } \mathbb{N}\}$.

(a) Show that 0 is the infimum of S .

(b) Show that S has no minimum.

(c) Does S have a maximum? Justify your answer.

2. For each of the following statements, determine whether it is true or false. If it is true, prove it; if it is false, exhibit a counterexample.

(a) If the sequence $\{|a_n|\}$ converges, then the sequence $\{a_n\}$ converges.

(b) If the sequence $\{a_n\}$ converges, then the sequence $\{|a_n|\}$ converges.

3. A sequence $\{a_n\}$ converges to infinity (written $\lim_{n \rightarrow \infty} a_n = \infty$) if and only if for each positive number c there is an index N such that $a_n > c$ for all indices $n \geq N$.

Given a sequence $\{a_n\}$ of positive real numbers, show that

$$\lim_{n \rightarrow \infty} a_n = \infty \quad \text{if and only if} \quad \lim_{n \rightarrow \infty} \frac{1}{a_n} = 0.$$

4. Consider the sequence $\{a_n\}$ defined as follows:

$$a_n = \begin{cases} 2 & \text{if } n = 1, \\ \frac{1}{2} \left(a_{n-1} + \frac{3}{a_{n-1}} \right) & \text{if } n > 1. \end{cases}$$

(a) Prove that $\{a_n\}$ is bounded below by $\sqrt{3}$. (Hint: show that $a_n^2 > 3$ for every index n .)

(b) Prove that $\{a_n\}$ is monotonically decreasing.

(c) By the monotone convergence theorem, $\{a_n\}$ converges to some real number a . What is a ? Justify your answer.