Name: ______________________________ **PID:** ________________

Math 142A
Midterm Exam 2
February 25, 2011

Turn off and put away your cell phone.
No calculators or any other electronic devices are allowed during this exam.
You may use one page of notes, but no books or other assistance on this exam.
Read each question carefully, answer each question completely, and show all of your work.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

#	Points	Score
1	6	
2	6	
3	6	
4	6	
Σ	24	

1. For each of the following *false* statements, exhibit a counterexample. Be sure to *briefly* state why each counterexample shows the corresponding statement to be false.

   (a) If the function $f + g : \mathbb{R} \to \mathbb{R}$ is continuous, then so are the functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$.

   (b) Every (not necessarily continuous) function $f : [0, 1] \to \mathbb{R}$ has a maximum.

   (c) Every continuous function $f : (0, 1) \to \mathbb{R}$ has a bounded image.

2. Let S be a sequentially compact set and let $f : S \to \mathbb{R}$ be a bounded continuous function. Prove that f attains its maximum.

3. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is continuous and that its image $f(\mathbb{R})$ is bounded. Prove that there is a solution $x \in \mathbb{R}$ to the equation $f(x) = x$.

4. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $\lim_{h \to 0} [f(x+h) - f(x-h)] = 0$ for every $x \in \mathbb{R}$. Must such a function f be continuous? If so, provide a proof; if not, exhibit a counterexample.