

Name: _____ PID: _____

TA: _____ Sec. No: _____ Sec. Time: _____

Math 20A.
Final Examination
December 8, 2010

Turn off and put away your cell phone.

No calculators or any other electronic devices are allowed during this exam.

You may use one page of notes, but no books or other assistance during this exam.

Read each question carefully, and answer each question completely.

Show all of your work; no credit will be given for unsupported answers.

Write your solutions clearly and legibly; no credit will be given for illegible solutions.

If any question is not clear, ask for clarification.

#	Points	Score
1	6	
2	6	
3	8	
4	6	
5	6	
6	6	
7	8	
8	6	
Σ	52	

1. (6 points) Find the each of the following limits, or state that it does not exist.

(a) $\lim_{x \rightarrow 4} \frac{x - 4}{\sqrt{x} - 2}$

(b) $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$

(c) $\lim_{x \rightarrow \infty} \frac{5x^4 - 6x^2 + 3}{3x^4 - x}$

2. (6 points) Find the derivative of the following functions. You need not simplify the resulting expressions.

(a) $f(x) = \sin^2(\cos(5x))$

(b) $F(x) = \int_{-1}^{2x^2} \sin^2(5\theta) d\theta$

(c) $g(x) = (x^2 - 1)^2(2x^3 - 5x)$

3. (8 points) If $\int_0^1 f(x) dx = 5$, $\int_0^2 f(x) dx = 2$, and $\int_0^2 g(x) dx = -3$, find

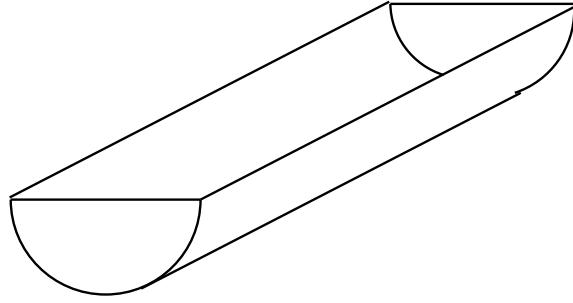
(a) $\int_1^2 f(x) dx$

(b) $\int_0^2 3f(u) du$

(c) $\int_1^0 f(x) dx$

(d) $\int_0^2 \{2g(x) - 3f(x)\} dx$

4. (6 points) A metal water trough with equal semicircular ends and open top needs to have a capacity of 64π cubic feet. Determine its radius r and length h if the trough is to require the least material for its construction.



5. (6 points) A spherical balloon is being inflated at the rate of 12 cubic inches per minute. What is the radius of the balloon when the rate of change of its surface area is 3 square inches per minute? (Note: the volume V and surface area A of a sphere of radius r are given by the formulas $V = \frac{4}{3}\pi r^3$ and $A = 4\pi r^2$)

6. (6 points) Calculate the following definite integrals.

(a) $\int_{-3}^4 |x^2 - 4| dx$

(b) $\int_{-3}^{-1} \frac{3 + 2x^2}{x} dx$

7. (8 points) Consider the graph of $f(x) = \frac{1}{1-x^2}$.

(a) Determine the vertical asymptote(s), if any.

(b) Determine the horizontal asymptote(s), if any.

(c) Determine the interval(s) of increase and the interval(s) of decrease.

(d) Determine the intervals of concavity.

8. (6 points) Find the linear approximation to $f(x) = x^{\frac{3}{2}}$ at $x = 25$ and use it to estimate $(25.06)^{\frac{3}{2}}$. (Note: $25^{\frac{3}{2}} = 125$.)