Math 20A **Final Examination v. A** **Name:** _____
December 12, 2017 **(Total Points: *50*)** **PID:** _____

Instructions

1. **Write your Name and PID in the spaces provided above.**
2. **Make sure your Name is on every page.**
3. **No calculators, tablets, phones, or other electronic devices are allowed during this exam.**
4. **Put away ANY devices that can be used for communication or can access the Internet.**
5. **You may use one handwritten page of notes, but no books or other assistance during this exam.**
6. **Read each question carefully and answer each question completely.**
7. **Write your solutions clearly in the spaces provided.**
8. **Show all of your work. No credit will be given for unsupported answers, even if correct.**

(2 points) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(6 points) 1. Find the points (x, y) on the graph $y = x^2$ that are closest to the point $(0, 2)$.

(6 points) 2. A 17-foot ladder is leaning against a wall. The bottom of the ladder slides away from the wall at 2 feet per second. How fast is the area of the triangle formed by the ladder, wall, and floor changing when the bottom of the ladder is 8 feet from the wall? (Note: $8^2 = 64$, $15^2 = 225$, and $17^2 = 289$; thus, $8^2 + 15^2 = 17^2$.)

(8 points) 3. $f(x) = x^3 - 6x + 5$

(a) Find the interval(s) where f is increasing and the intervals where f is decreasing.

(b) Find the local maximum and the local minimum value(s) of f.

(c) Find the intervals where the graph of f is concave up and the intervals where the graph of f is concave down.

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(8 points) 4. Evaluate the following limits.

(a) $\lim_{x \to 1} \dfrac{x^2 - 1}{x^3 + 3}$

(b) $\lim_{x \to \infty} \sqrt{x+1} - \sqrt{x}$

(c) $\lim_{x \to 0^+} \sqrt{x}\, \ln(2x)$

(d) $\lim_{x \to 0} \dfrac{\int_0^x \sqrt{1+t^2}\, dt}{x}$

(4 points) 5. Let $f(x) = \ln(x) + \tan^{-1}(x)$ and $g(x) = f^{-1}(x)$. Evaluate the derivative $g'\left(\ln(2) + \tan^{-1}(2)\right)$.

(4 points) 6. The following table gives the values of a function obtained from an experiment. Estimate $\int_0^8 f(x)\,dx$ with the Riemann sum using 4 subintervals and right endpoints as sample points. (Note: You need not do the arithmetic; you may leave your answer in the form of a sum.)

x	0	2	4	6	8
$f(x)$	8.3	8.0	7.3	5.5	1.3

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(6 points) 7. Evaluate the following limits.

(a) $\lim_{x \to 0^+} \int_x^1 \frac{1}{\sqrt{t}}\, dt$

(b) $\lim_{x \to 0^+} \int_x^1 \frac{1}{t}\, dt$

(6 points) 8. The Fresnel Sine Function $S(x)$ is defined by

$$S(x) = \int_0^x \sin\left(\frac{\pi}{2}t^2\right)\, dt.$$

Find the critical point of S in the interval $(1, 2)$ and determine whether S has a local maximum or a local minimum value at that point.