

Math 142B
September 5, 2018

Question 1 Given a function $f : [a, b] \rightarrow \mathbb{R}$. Then,

- A. if f is bounded, then f is continuous.
- B. if f is monotonically increasing then f is continuous.
- C. if f is not continuous, then f is not integrable.
- D. if f is integrable, then f is continuous.
- *E. None of the above: all the above statements are false.

Question 2 The function $f : \mathbb{R} \rightarrow \mathbb{R}$ given by

$f(x) = \begin{cases} 0 & \text{if } x = 0 \\ e^{-\frac{1}{x^2}} & \text{if } x \neq 0 \end{cases}$ is an example of a function that is

- A. bounded.
- B. infinitely differentiable.
- C. analytic.
- *D. **A** and **B**.
- E. **B** and **C**.

Question 3 Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be the exponential function $f(x) = e^x$.
Then,

A. The n^{th} Taylor polynomial for f at $x = 0$ is

$$p_n(x) = \sum_{k=0}^n \frac{1}{k!} x^k = 1 + \frac{1}{2} + \cdots + \frac{1}{n!} x^n.$$

B. $f^{(k)}(0) = p_n^{(k)}(0)$ for $0 \leq k \leq n$.

C. $\lim_{n \rightarrow \infty} p_n(x) = f(x)$ for every x .

D. **A** and **B**.

*E. **A**, **B**, and **C**.

Question 4 Given a series $\sum_{k=1}^{\infty} a_k$ such that $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L$. Then,

- A. $\sum_{k=1}^{\infty} a_k$ converges conditionally whenever $L = 1$.
- B. $\sum_{k=1}^{\infty} a_k$ converges absolutely whenever $L < 1$.
- C. $\sum_{k=1}^{\infty} a_k$ diverges whenever $L > 1$.
- D. **A** and **B**.
- *E. **B** and **C**.

Question 5 Setting $f_n(x) = \sum_{k=0}^n (-1)^k x^{2k}$ for $x \in [0, 1]$ defines a sequence $\{f_n : [0, 1] \rightarrow \mathbb{R}\}$.

Let $f : [0, 1] \rightarrow \mathbb{R}$ be defined by $f(x) = \frac{1}{1+x^2}$. Then,

- *A. $\int_0^1 f = \lim_{n \rightarrow \infty} \int_0^1 f_n$; this is the Newton-Gregory Formula.
- B. $\{f_n\}$ converges pointwise on $[0, 1]$ to f .
- C. $\{f_n\}$ converges uniformly on $[0, 1]$ to f .
- D. All of the above.
- E. None of the above.

Question 6 Setting $f_n(x) = \frac{1}{n} \tan^{-1}(n^2x)$ defines a sequence $\{f_n : \mathbb{R} \rightarrow \mathbb{R}\}$ of infinitely differentiable functions. In addition,

- A. $\{f_n\}$ converges pointwise on \mathbb{R} to 0 since $\lim_{n \rightarrow \infty} \frac{1}{n} \tan^{-1}(n^2x) = 0$ for all x .
- B. $\{f_n\}$ converges uniformly on \mathbb{R} to 0 since $|f_n(x)| < \frac{\pi}{2n}$ for all x .
- C. $\{f'_n(0)\}$ is unbounded, even though $\{f_n\}$ converges uniformly on \mathbb{R} .
- D. **A** and **B**.
- *E. **A**, **B**, and **C**.

Question 7 Clicker questions have been used throughout this course to think about the ideas in new ways and to review previously considered ideas.

Should we look at an extended list of clicker questions during our review day tomorrow?

- A. Yes! Clicker questions provide a fun and engaging way to review.
- B. No! I'd be happiest if I never saw another clicker question in my life.
- C. What are clicker questions? Could they be causing my tinnitus?
- D. All of the above.
- E. None of the above.