Math 142B
September 6, 2018

Question 1 Setting $f_n(x) = \sum_{k=0}^{n} (-1)^k x^{2k}$ for $x \in [0,1]$ defines a sequence $\{f_n : [0,1] \to \mathbb{R}\}$.

Let $f : [0,1] \to \mathbb{R}$ be defined by $f(x) = \frac{1}{1+x^2}$. Then,

- *A. $\int_0^1 f = \lim_{n\to\infty} \int_0^1 f_n$; this is the Newton-Gregory Formula.
- B. $\{f_n\}$ converges pointwise on $[0,1]$ to f.
- C. $\{f_n\}$ converges uniformly on $[0,1]$ to f.
- D. All of the above.
- E. None of the above.

Question 2 Given a series $\sum_{k=1}^{\infty} a_k$ such that $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L$. Then,

A. $\sum_{k=1}^{\infty} a_k$ converges conditionally whenever $L = 1$.

B. $\sum_{k=1}^{\infty} a_k$ converges absolutely whenever $L < 1$.

C. $\sum_{k=1}^{\infty} a_k$ diverges whenever $L > 1$.

D. **A** and **B**.

*E. **B** and **C**.

Question 3 Setting $f_n(x) = \dfrac{1}{n}\tan^{-1}\left(n^2 x\right)$ defines a sequence $\{f_n : \mathbb{R} \to \mathbb{R}\}$ of infinitely differentiable functions. In addition,

A. $\{f_n\}$ converges pointwise on $\mathbb{R}$ to 0 since $\lim_{n\to\infty} \dfrac{1}{n}\tan^{-1}(n^2 x) = 0$ for all x.

B. $\{f_n\}$ converges uniformly on $\mathbb{R}$ to 0 since $|f_n(x)| < \frac{\pi}{2n}$ for all x.

C. $\{f_n'(0)\}$ is unbounded, even though $\{f_n\}$ converges uniformly on $\mathbb{R}$.

D. **A** and **B**.

*E. **A**, **B**, and **C**.

Question 4 Given a power series $\sum_{k=0}^{\infty} c_k x^k$ with D its domain of convergence. Let $r > 0$ such that $(-r, r) \subseteq D$ and define $f : (-r, r) \to \mathbb{R}$ by $f(x) = \sum_{k=0}^{\infty} c_k x^k$. Then,

A. f is infinitely differentiable.

B. $\frac{d^n}{dx^n} f(x) = \sum_{k=0}^{\infty} c_k \frac{d^n}{dx^n} x^k$ for each index n.

C. $\frac{f^{(n)}(0)}{n!} = c_n$ for each index n.

*D. All of the above.

E. None of the above.

Question 5 Given a power series $\sum_{k=0}^{\infty} c_k x^k$ with a bounded domain of convergence D. Define $R := \sup D$. Then,

A. $(-R, R) \subseteq D \subseteq [-R, R]$.

B. D is one of: $(-R, R)$, $[-R, R)$, $(-R, R]$, or $[-R, R]$.

C. R is called the radius of convergence of $\sum_{k=0}^{\infty} c_k x^k$.

*D. All of the above.

E. None of the above.

Question 6 A sequence of functions $\{f_k\}$ is uniformly Cauchy on D if for every $\varepsilon > 0$, there is an index N such that $|f_m(x) - f_n(x)| < \varepsilon$ for all indices $m, n \geq N$ and all $x \in D$.
Given that $\{g_k\}$ is uniformly Cauchy on $[0, 1]$. Then,

A. For each $x \in [0, 1]$, $\{g_k(x)\}$ is a Cauchy sequence.

B. $\{g_k\}$ converges pointwise to a function $g : [0, 1] \to \mathbb{R}$.

C. $\{g_k\}$ converges uniformly to a function $g : [0, 1] \to \mathbb{R}$.

D. **A** and **B**.

*E. **A**, **B**, and **C**.

Question 7 Clicker questions were used throughout this course to think about the ideas in new ways and to review previously considered ideas.
I found the clicker questions to be

A. a helpful aid for study and review.

B. an amusing way to begin each class.

C. a complete waste of time; I would have rather slept longer.

D. each of the above, depending on which day it was.

E. none of the above; I think I'm getting tinnitus.