Math 142B
August 8, 2018

Question 1 Given P a partition of $[a, b]$. P^* is a refinement of P if

A. $P^* \supseteq P$.

B. P^* is a partition of P.

C. P^* is a partition of $[a, b]$.

D. **A** and **B**.

*E. **A** and **C**.

Question 2 Given $f : [a, b] \to \mathbb{R}$ bounded, P a partition of $[a, b]$, and P^* a refinement of P. The refinement lemma asserts that

*A. $L(f, P) \leq L(f, P^*) \leq U(f, P^*) \leq U(f, P)$.

B. $L(f, P^*) \leq L(f, P) \leq U(f, P) \leq U(f, P^*)$.

C. $L(f, P) \leq L(f, P^*)$ and $U(f, P) \leq U(f, P^*)$.

D. $L(f, P^*) \leq L(f, P)$ and $U(f, P^*) \leq U(f, P)$.

E. None of the above.

Question 3 Given $P = \{x_0, x_1, \ldots, x_{n-1}, x_n\}$ a partition of $[a, b]$. The gap of P is defined by:

*A. $\text{gap}(P) = \max_{1 \leq i \leq n} (x_i - x_{i-1})$.

B. $\text{gap}(P) = \min_{1 \leq i \leq n} (x_i - x_{i-1})$.

C. $\text{gap}(P) = \frac{1}{n} \sum_{i=1}^{n} (x_i - x_{i-1})$.

D. $\text{gap}(P) = \frac{b-a}{n}$.

E. **C** and **D**; they are the same.

Question 4 Given $f : [a, b] \to \mathbb{R}$.

A. $\underline{\int_a^b} f$ and $\overline{\int_a^b} f$ both exist if and only if f is bounded.

B. $\underline{\int_a^b} f$ is undefined if f is not bounded below.

C. $\overline{\int_a^b} f$ is undefined if f is not bounded above.

*D. **A**, **B**, and **C**.

E. None of the above: $\underline{\int_a^b} f$ and $\overline{\int_a^b} f$ always exist.

Question 5 Recall Dirichlet's function $f : [0, 1] \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} 0 & \text{if } x \in \mathbb{Q}, \\ 1 & \text{if } x \in \mathbb{R} \setminus \mathbb{Q}. \end{cases}$$

A. f is integrable because $\underline{\int_a^b} f$ and $\overline{\int_a^b} f$ both exist.

B. f is integrable because f is bounded.

C. **A** and **B**; they are equivalent.

D. f is not integrable because f is not continuous.

*E. f is not integrable because $\underline{\int_a^b} f \neq \overline{\int_a^b} f$.