Math 142B
August 9, 2018

Question 1 A function $f : [a, b] \to \mathbb{R}$ is uniformly continuous provided that

*A. whenever $\{u_n\}$ and $\{v_n\}$ are sequences in $[a, b]$ such that $\lim_{n\to\infty} [u_n - v_n] = 0$, then $\lim_{n\to\infty} [f(u_n) - f(v_n)] = 0$.

B. whenever $\{u_n\}$ is a sequence in $[a, b]$ such that $\lim_{n\to\infty} u_n = u_0$, then $\lim_{n\to\infty} f(u_n) = f(u_0)$.

C. whenever $\{u_n\}$ is a sequence in $[a, b]$ such that $\lim_{n\to\infty} u_n = u_0$, then $\lim_{n\to\infty} \frac{f(u_n) - f(u_0)}{u_n - u_0} = f'(u_0)$.

D. All of the above.

E. None of the above. Continuity is never uniform.

Question 2 The proof that $\int_a^b \alpha f = \alpha \int_a^b f$ uses the fact that for $\alpha < 0$ and any partition P of $[a, b]$,

$$U(\alpha f, P) = \alpha L(f, P) \quad \text{and} \quad L(\alpha f, P) = \alpha U(f, P).$$

This is true because, on any interval I,

A. $\sup\{-f(x) \mid x \in I\} = -\inf\{f(x) \mid x \in I\}$.

B. $\inf\{-f(x) \mid x \in I\} = -\sup\{f(x) \mid x \in I\}$.

C. $\sup\{\alpha f(x) \mid x \in I\} = \alpha \inf\{f(x) \mid x \in I\}$ for all $\alpha < 0$.

D. $\inf\{\alpha f(x) \mid x \in I\} = \alpha \sup\{f(x) \mid x \in I\}$ for all $\alpha < 0$.

*E. All of the above.

Question 3 The proof that $\int_a^b (f+g) = \int_a^b f + \int_a^b g$ uses the fact that $U(f+g,P) \le U(f,P) + U(g,P)$ for any partition P of $[a,b]$. This is true because, on any interval I,

A. $f(x) + g(x) \le \sup_{x\in I} f + \sup_{x\in I} g$.

B. $\sup_{x\in I}(f+g) \le \sup_{x\in I} f + \sup_{x\in I} g$.

*C. **A** and **B**; after all, **A** implies **B**.

D. $\sup_{x\in I}(-f) = -\inf_{x\in I} f$.

E. None of the above.

Question 4 The Archimedes-Riemann Theorem asserts that $f : [a, b] \to \mathbb{R}$ is integrable if and only if

*A. there is a sequence of partitions $\{P_n\}$ of $[a, b]$ with $\lim_{n\to\infty} [U(f, P_n) - L(f, P_n)] = 0$.

B. $\underline{\int_a^b} f = \overline{\int_a^b} f$.

C. f is monotonically increasing or monotonically decreasing on $[a, b]$.

D. f is continuous on $[a, b]$.

E. All of the above.

Question 5 Given $f : [a, b] \to \mathbb{R}$ bounded, P a partition of $[a, b]$, and P^* a refinement of P. The refinement lemma asserts that

A. $L(f, P^*) \le L(f, P)$ and $U(f, P^*) \le U(f, P)$.

B. $L(f, P) \le L(f, P^*)$ and $U(f, P) \le U(f, P^*)$.

C. $L(f, P^*) \le L(f, P) \le U(f, P) \le U(f, P^*)$.

*D. $L(f, P) \le L(f, P^*) \le U(f, P^*) \le U(f, P)$.

E. None of the above.