Math 142B
August 13, 2018

Question 1 Given $f : [a, b] \to \mathbb{R}$ bounded. Let $\alpha := \inf_{x \in [a,b]} \{f(x)\}$. Which of the following statements are false?

*A. There is a point $x_0 \in [a, b]$ at which $f(x_0) = \alpha$.

B. $\alpha \cdot [b - a] \leq L(f, P)$ for every partition P of $[a, b]$.

C. $\alpha \cdot [b - a] \leq \underline{\int_a^b} f$.

D. None of the above; they're all true.

E. All of the above; they're all false.

Question 2 Given $f : [a, b] \to \mathbb{R}$ bounded. Let $\beta := \sup_{x \in [a,b]} \{f(x)\}$.

Which of the following statements are false?

A. $f(x) \leq \beta$ for every $x \in [a, b]$.

B. $U(f, P) \leq \beta \cdot [b - a]$ for every partition P of $[a, b]$.

C. $\overline{\int_a^b} f \leq \alpha \cdot [b - a]$.

*D. None of the above; they're all true.

E. All of the above; they're all false.

Question 3 Given $f : [a, b] \to \mathbb{R}$ continuous and $f : (a, b) \to \mathbb{R}$ differentiable. Then,

A. There is a point $x_0 \in (a, b)$ at which
$$f'(x_0) = \frac{f(b) - f(a)}{b - a}.$$

B. $f : [a, b] \to \mathbb{R}$ is integrable.

C. $f' : (a, b) \to \mathbb{R}$ is continuous.

*D. **A** and **B**.

E. **A**, **B**, and **C**.

Question 4 The Archimedes-Riemann Theorem asserts that $f : [a, b] \to \mathbb{R}$ is integrable if and only if

- *A. there is a sequence of partitions $\{P_n\}$ of $[a, b]$ with $\lim_{n\to\infty} [U(f, P_n) - L(f, P_n)] = 0$.
- B. $\underline{\int_a^b} f = \overline{\int_a^b} f$.
- C. f is monotonically increasing or monotonically decreasing on $[a, b]$.
- D. f is continuous on $[a, b]$.
- E. All of the above.

Question 5 A function $f : [a, b] \to \mathbb{R}$ is uniformly continuous provided that

*A. whenever $\{u_n\}$ and $\{v_n\}$ are sequences in $[a, b]$ such that $\lim_{n\to\infty} [u_n - v_n] = 0$, then $\lim_{n\to\infty} [f(u_n) - f(v_n)] = 0$.

B. whenever $\{u_n\}$ is a sequence in $[a, b]$ such that $\lim_{n\to\infty} u_n = u_0$, then $\lim_{n\to\infty} f(u_n) = f(u_0)$.

C. whenever $\{u_n\}$ is a sequence in $[a, b]$ such that $\lim_{n\to\infty} u_n = u_0$, then $\lim_{n\to\infty} \frac{f(u_n) - f(u_0)}{u_n - u_0} = f'(u_0)$.

D. All of the above.

E. None of the above. Continuity is never uniform.