Math 142B
August 14, 2018

Question 1 Given $f : [a, b] \to \mathbb{R}$ continuous on $[a, b]$ and differentiable on (a, b). Which of the following conditions imply that $\int_a^b f' = f(b) - f(a)$?

A. $f' : (a, b) \to \mathbb{R}$ is bounded.

B. $f' : (a, b) \to \mathbb{R}$ is bounded and continuous.

C. Every extension of f' to $[a, b]$ is integrable.

D. Both **A** and **C**; after all, **A** implies **C**.

*E. Both **B** and **C**; after all, **B** implies **C**.

Question 2 Consider the function $f : [0, 1] \to \mathbb{R}$ given by $f(x) = e^{-x^2}$. We can say that

A. f is integrable because f is continuous.

B. $\int_0^1 f = F(1) - F(0)$ for any function that satisfies $F'(x) = f(x)$.

C. f does not satisfy the hypothesis of the First Fundamental Theorem because there is no function $F : (0, 1) \to \mathbb{R}$ for which $F'(x) = f(x)$.

*D. **A** and **B**.

E. **A** and **C**.

Question 3 Given $f : [a, b] \to \mathbb{R}$ bounded and $\{P_n\}$ a sequence of partitions of $[a, b]$. Then,

A. $U(f, P_n) - L(f, P_n) \geq \overline{\int_a^b} f - \underline{\int_a^b} f \geq 0$ for every index n.

B. $\lim_{n\to\infty} [U(f, P_n) - L(f, P_n)] = 0$ implies f is integrable.

C. $\lim_{n\to\infty} [U(f, P_n) - L(f, P_n)] = 0$ implies $\lim_{n\to\infty} \text{gap}(P_n) = 0$.

*D. **A** and **B**.

E. **A**, **B**, and **C**.

Question 4 Given $f : [a, b] \to \mathbb{R}$ bounded and $\{P_n\}$ a sequence of partitions of $[a, b]$. If $\lim_{n\to\infty} \text{gap}\,(P_n) = 0$, then

A. P_{n+1} is a refinement of P_n for every index n.

B. $\{P_n\}$ is an Archimedean sequence of partitions for f on $[a, b]$.

C. f is integrable.

D. **B** and **C**; they are equivalent.

*E. None of the above.

Question 5 Given $f : [a, b] \to \mathbb{R}$ bounded and $\{P_n\}$ an Archimedean sequence of partitions for f on $[a, b]$. Then,

A. $\lim_{n\to\infty} [U(f, P_n) - L(f, P_n)] = 0$.

B. f is integrable.

C. $\lim_{n\to\infty} \text{gap}(P_n) = 0$.

*D. **A** and **B**.

E. **A**, **B**, and **C**.