Math 142B
August 15, 2018

Question 1 Given $f : [0,2] \to \mathbb{R}$ bounded. Let $P_0 = \{0,2\}$ and $P_1 = \{0,1,2\}$. Then,

A. P_1 is a refinement of P_0.

B. $L(f, P_0) \leq L(f, P_1)$ because $\inf_{x\in[0,2]} f(x) \leq \inf_{x\in[0,1]} f(x)$ and $\inf_{x\in[0,2]} f(x) \leq \inf_{x\in[1,2]} f(x)$.

C. $U(f, P_1) \leq U(f, P_0)$ because $\sup_{x\in[0,1]} f(x) \leq \sup_{x\in[0,2]} f(x)$ and $\sup_{x\in[1,2]} f(x) \leq \sup_{x\in[0,2]} f(x)$.

*D. All of the above.

E. None of the above.

Question 2 Given $f : [a, b] \to \mathbb{R}$ bounded. Then $\sup_{\text{partitions } P} L(f, P)$ is called

*A. $\underline{\int_a^b} f$, the lower integral of f on $[a, b]$.

B. $\overline{\int_a^b} f$, the upper integral of f on $[a, b]$.

C. $\int_a^b f$, the integral of f on $[a, b]$.

D. All of the above; they are the same when f is integrable.

E. None of the above; not much can be said when f is merely bounded.

Question 3 Given $f : [a, b] \to \mathbb{R}$ bounded and $\{P_n\}$ an Archimedean sequence of partitions for f on $[a, b]$. Then,

A. $\lim_{n\to\infty} U(f, P_n) = \inf_{\text{partitions } P} U(f, P)$.

B. $\lim_{n\to\infty} L(f, P_n) = \sup_{\text{partitions } P} L(f, P)$.

C. $\underline{\int_a^b} f = \overline{\int_a^b} f$.

D. f is integrable.

*E. All of the above.

Question 4 Given $f : [a, b] \to \mathbb{R}$ monotone; that is, monotonically increasing or monotonically decreasing. We can say that

A. f is bounded.

B. f is integrable.

C. The sequence $\{P_n\}$ of the n^{th} regular partitions of $[a, b]$ form an Archimedean sequence of partitions for f.

D. **A** and **B**.

*E. **A**, **B**, and **C**.

Question 5 Given $f : [a, b] \to \mathbb{R}$ continuous on $[a, b]$ and differentiable on (a, b). Which of the following conditions imply that
$$\int_a^b f' = f(b) - f(a)?$$

A. $f' : (a, b) \to \mathbb{R}$ is bounded.

B. $f' : (a, b) \to \mathbb{R}$ is bounded and continuous.

C. Every extension of f' to $[a, b]$ is integrable.

D. Both **A** and **C**; after all, **A** implies **C**.

*E. Both **B** and **C**; after all, **B** implies **C**.

Question 6 Given $f : [a, b] \to \mathbb{R}$ bounded and $\{P_n\}$ a sequence of partitions of $[a, b]$. Then,

A. $U(f, P_n) - L(f, P_n) \geq \overline{\int_a^b} f - \underline{\int_a^b} f \geq 0$ for every index n.

B. $\lim_{n\to\infty} [U(f, P_n) - L(f, P_n)] = 0$ implies f is integrable.

C. $\lim_{n\to\infty} [U(f, P_n) - L(f, P_n)] = 0$ implies $\lim_{n\to\infty} \text{gap}(P_n) = 0$.

*D. **A** and **B**.

E. **A**, **B**, and **C**.

Question 7 Given $f : [a, b] \to \mathbb{R}$ bounded and $\{P_n\}$ a sequence of partitions of $[a, b]$. If $\lim_{n\to\infty} \text{gap}(P_n) = 0$, then

A. P_{n+1} is a refinement of P_n for every index n.

B. $\{P_n\}$ is an Archimedean sequence of partitions for f on $[a, b]$.

C. f is integrable.

D. **B** and **C**; they are equivalent.

*E. None of the above.

Question 8 Given $f : [a, b] \to \mathbb{R}$ bounded and $\{P_n\}$ an Archimedean sequence of partitions for f on $[a, b]$. Then,

A. $\lim_{n\to\infty} [U(f, P_n) - L(f, P_n)] = 0.$

B. f is integrable.

C. $\lim_{n\to\infty} \text{gap}(P_n) = 0.$

*D. **A** and **B**.

E. **A**, **B**, and **C**.