Math 142B
August 20, 2018

Question 1 Rolle's Theorem asserts that if $f : [a, b] \to \mathbb{R}$ is continuous with $f(a) = f(b)$ and its restriction $f : (a, b) \to \mathbb{R}$ is differentiable, then there is a point $x_0 \in (a, b)$ at which $f'(x_0) = 0$.
Which of the following statements does the proof of Rolle's Theorem depend on?

A. f attains both its extreme values (maximum and minimum) on $[a, b]$.

B. If f attains an extreme value at $x_0 \in (a, b)$, then $f'(x_0) = 0$.

C. If f attains its extreme values at both a and b, then f is the constant function identically equal to $f(a) = f(b)$.

*D. **A**, **B**, and **C**.

*E. **A** and **B**; **C** is not necessary since the derivative of a constant function is 0 for all x.

[Note: **D** is perhaps the best choice, but an argument can be made for **E**.]

Question 2 The Mean Value Theorem asserts that if $f : [a, b] \to \mathbb{R}$ is continuous and its restriction $f : (a, b) \to \mathbb{R}$ is differentiable, then there is a point $x_0 \in (a, b)$ at which $f'(x_0) = \dfrac{f(b) - f(a)}{b - a}$.
The Mean Value Theorem can be proved by

*A. applying Rolle's Theorem to $g(x) = f(x) - f(a) - m(x - a)$ with $m := \frac{f(b)-f(a)}{b-a}$.

B. applying the Extreme Value Theorem to find a point $x_0 \in (a, b)$ at which $f'(x_0)$ has an extreme value.

C. applying the Mean Value Theorem for integrals to find x_0 so that $f(x_0) = \dfrac{1}{b - a} \displaystyle\int_a^b f$.

D. All of the above; **A**, **B**, and **C** are all part of the proof of the Mean Value Theorem.

E. None of the above; the Mean Value Theorem only applies to differentiable functions $f : [a, b] \to \mathbb{R}$.

Question 3 The Cauchy Mean Value Theorem asserts that if $f, g : [a, b] \to \mathbb{R}$ are continuous and their restrictions $f, g : (a, b) \to \mathbb{R}$ are differentiable with $g'(x) \neq 0$ for all $x \in (a, b)$, then there is a point $x_0 \in (a, b)$ at which $f'(x_0) = \frac{f(b)-f(a)}{g(b)-g(a)}$.
The Cauchy Mean Value Theorem can be proved by

*A. applying Rolle's Theorem to $h(x) = f(x) - mg(x)$ with the constant $m := \frac{f(b)-f(a)}{g(b)-g(a)}$.

B. applying the Mean Value Theorem to $h(x) = \frac{f(x)-f(a)}{g(x)-g(a)}$.

C. applying the Extreme Value Theorem to $h(x) = \frac{f(x)}{g(x)}$ in order to find a point x_0 at which $h'(x_0) = 0$.

D. all of the above; the Cauchy Mean Value Theorem can be proved in many ways.

E. none of the above; Cauchy had nothing to do with the Mean Value Theorem.

Question 4 Given a function $f : [a, b] \to \mathbb{R}$. We can say that

A. if f is bounded, then f must have both a maximum and a minimum.

B. if f is bounded, then f must have either a maximum or a minimum.

C. if f is a constant function, then f has neither a maximum nor a minimum.

*D. f need not attain a maximum or a minimum.

E. **C** and **D**.

Question 5 Which of the following functions are properly defined?

A. $f : [0,1] \to \mathbb{R}$ given by $f(x) = \sin\left(\frac{\pi}{1-x}\right)$.

B. $f : [0,1] \to \mathbb{R}$ given by $f(x) = \log(x)$.

C. $f : [0,1] \to \mathbb{R}$ given by $f(x) = \tan(\pi x)$.

D. All of the above; they are all familiar functions from calculus.

*E. None of the above: These functions are all pure balderdash; that is, they are pure nonsense.

Question 6 Given a function $f : [a, b] \to \mathbb{R}$. Then,

A. if f is bounded, then f is continuous.

B. if f is monotonically increasing then f is continuous.

C. if f is not continuous, then f is not integrable.

D. if f is integrable, then f is continuous.

*E. None of the above: all the above statements are pure malarkey; that is, they are pure nonsense.