Math 142B
August 23, 2018

Question 1 We can write $f(x) = \log(1+x)$ in the form $f(x) = \sum_{k=1}^{n} \frac{(-1)^{k+1}}{k} x^k + R_n(x)$. We can say that

A. $\lim_{n\to\infty} R_n(x) = 0$.

B. $R_n(x) = \frac{(-1)^n}{k(1+c)^{n+1}} x^{n+1}$ for some c strictly between 0 and x.

C. $R_n(x) = \int_1^{1+x} \frac{(1-t)^n}{t}\, dt$.

D. All of the above.

*E. All of the above, provided $-1 < x \leq 1$; otherwise, **A** is false.

Question 2 Let I be a neighborhood of x_0 and let $f : I \to \mathbb{R}$ be a function with $n+1$ derivatives. Then,

$$f(x_0+h) = \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!} h^k + R_n(x),$$

where the n^{th} remainder $R_n(x)$ is equal to

A. $\dfrac{f^{(n+1)}(a)}{(n+1)!} h^{n+1}$ for some a strictly between x_0 and x_0+h.

B. $\dfrac{f^{(n+1)}(x_0+b)}{(n+1)!} h^{n+1}$ for some b strictly between 0 and h.

C. $\dfrac{f^{(n+1)}(c)}{(n+1)!} h^{n+1}$ for some c strictly between x and x_0.

*D. **A** or **B**.

E. **A**, **B**, or **C**.

[Note: **E** would be true if x were defined to be $x := x_0 + h$.]

Question 3 A bounded function $f : [a, b] \to \mathbb{R}$ is integrable if and only if

A. $\underline{\int_a^b} f = \overline{\int_a^b} f$.

B. there is a sequence of partitions $\{P_n\}$ of $[a, b]$ with $\lim_{n\to\infty} [U(f, P_n) - L(f, P_n)] = 0$.

C. for every $\varepsilon > 0$ there is a corresponding partition P of $[a, b]$ for which $[U(f, P) - L(f, P)] < \varepsilon$

*D. All of the above; they are equivalent.

E. None of the above; not all bounded functions are integrable.

Question 4 Given $x_0 \in \mathbb{R}$ and $f : \mathbb{R} \to \mathbb{R}$ with derivatives of all orders. Then $p_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k$ is the n^{th} Taylor polynomial for f at $x = x_0$, and

A. p_n has contact of order n with f at x_0.

B. $f(x) - p_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x - x_0)^{n+1}$ for some c strictly between x and x_0.

C. $\lim_{n\to\infty} p_n(x) = f(x)$ for every x.

*D. **A** and **B**.

E. **A**, **B**, and **C**.

Question 5 Given $f : [a, b] \to \mathbb{R}$ bounded and $\{P_n\}$ a sequence of partitions of $[a, b]$. Then,

A. $U(f, P_n) - L(f, P_n) \geq \overline{\int_a^b} f - \underline{\int_a^b} f \geq 0$ for every index n.

B. $\lim_{n\to\infty} [U(f, P_n) - L(f, P_n)] = 0$ implies f is integrable.

C. $\lim_{n\to\infty} [U(f, P_n) - L(f, P_n)] = 0$ implies $\lim_{n\to\infty} \text{gap}(P_n) = 0$.

*D. **A** and **B**.

E. **A**, **B**, and **C**.

Question 6 Given $f : [a, b] \to \mathbb{R}$ bounded and $P \subseteq P^*$ partitions of $[a, b]$.

A. P^* is a refinement of P.

B. $L(f, P^*) = L(f, P)$ and $U(f, P^*) = U(f, P)$.

C. $L(f, P^*) \geq L(f, P)$ and $U(f, P^*) \leq U(f, P)$.

D. **A** and **B**.

*E. **A** and **C**.