Math 142B
August 27, 2018

Question 1 Given a neighborhood I of a point x_0 and an infinitely differentiable function $f : I \to \mathbb{R}$. The First Fundamental Theorem asserts that $f(x) = f(x_0) + \int_{x_0}^{x} f'(t)\, dt$. Then,

A. $\int_{x_0}^{x} f'(t)\, dt = f'(x_0)(x - x_0) + \int_{x_0}^{x} f''(t)(x - t)\, dt$, after an integration by parts.

B. $\int_{x_0}^{x} f''(t)(x - t)\, dt$ is the 1^{st} Cauchy integral remainder for f at $x = x_0$.

C. $f(x_0) + f'(x_0)(x - x_0)$ is the 1^{st} Taylor polynomial for f at $x = x_0$.

D. **B** and **C**.

*E. **A**, **B**, and **C**.

Question 2 Given a neighborhood I of a point x_0 and a function $f : I \to \mathbb{R}$ with $n+1$ derivatives. Let $p_n(x)$ be the n^{th} Taylor polynomial for f at $x = x_0$. Then,

A. There is a c strictly between x and x_0 at which
$$f(x) - p_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x - x_0)^{n+1}.$$

B. There is a c strictly between x and x_0 at which
$$\frac{f^{(n+1)}(c)}{(n+1)!}(x - x_0)^{n+1} = \frac{1}{n!}\int_{x_0}^{x} f^{(n+1)}(t)(x-t)^n\,dt.$$

C. $\lim_{n\to\infty} \left\{ \frac{1}{n!}\int_{x_0}^{x} f^{(n+1)}(t)(x-t)^n\,dt \right\} = 0.$

*D. **A** and **B**.

E. **A**, **B**, and **C**.

Question 3 Given any number β. Define $f : (-1, 1) \to \mathbb{R}$ by $f(x) = (1 + x)^\beta$. Then,

*A. $f^{(k)}(x) = \beta(\beta - 1) \cdots (\beta - k + 1)(1 + x)^{\beta - k}$.

B. $f(x) = \sum_{k=0}^{\infty} \frac{\beta(\beta - 1) \cdots (\beta - k + 1)}{k!}(1 + x)^{\beta - k}$.

C. $f(x) = \sum_{k=0}^{\infty} \binom{\beta}{k}(1 + x)^{\beta - k}$.

D. **B** and **C**; they are the same since $\binom{\beta}{k} = \frac{\beta(\beta-1)\cdots(\beta-k+1)}{k!}$.

E. All of the above.

Question 4 Given $x_0 \in \mathbb{R}$ and $f : \mathbb{R} \to \mathbb{R}$ with derivatives of all orders. Then $p_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k$ is the n^{th} Taylor polynomial for f at $x = x_0$, and

A. p_n has contact of order n with f at x_0.

B. $f(x) - p_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x - x_0)^{n+1}$ for some c strictly between x and x_0.

C. $\lim_{n\to\infty} p_n(x) = f(x)$ for every x.

*D. **A** and **B**.

E. **A**, **B**, and **C**.

Question 5 Given a neighborhood I of a point x_0 and an infinitely differentiable function $f : I \to \mathbb{R}$. Then, $f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k$ for every $x \in I$ whenever

A. $\lim_{n\to\infty} [f(x) - p_n(x)] \to 0$ for every $x \in I$, where p_n is the n^{th} Taylor polynomial for f at x_0.

B. $\lim_{n\to\infty} \frac{f^{(n+1)}(x)}{(n+1)!}(x - x_0)^{n+1} = 0$ for every $x \in I$.

C. There is an $M > 0$ for which $\left| f^{(k)}(x) \right| \leq M^k$ for every $x \in I$ and every index k.

D. **A** and **B**.

*E. **A** and **C**.

[Note: Whether or not **B** is true is a Math 142B open question.]

Question 6 Given a function $f : [a, b] \to \mathbb{R}$. Then,

A. if f is bounded, then f is continuous.

B. if f is monotonically increasing then f is continuous.

C. if f is not continuous, then f is not integrable.

D. if f is integrable, then f is continuous.

*E. None of the above: all the above statements are false.