Math 142B
Midterm Exam 1
August 14, 2014
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Instructions

1. **You may use any type of calculator, but no other electronic devices during this exam.**
2. **You may use one page of notes, but no books or other assistance during this exam.**
3. **Write your *Name*, *PID*, and *Section* on the front of your Blue Book.**
4. **Write your solutions clearly in your Blue Book**
   (a) **Carefully indicate the number and letter of each question and question part.**
   (b) **Present your answers in the same order they appear in the exam.**
   (c) **Start each question on a new side of a page.**
5. **Read each question carefully, and answer each question completely.**
6. **Show all of your work; no credit will be given for unsupported answers.**

1. Let $f : [0, 1] \to \mathbb{R}$ be given by
$$f(x) = \begin{cases} x & \text{if } x \text{ is rational,} \\ 1 & \text{if } x \text{ is irrational.} \end{cases}$$
   (a) Show that the lower integral $\underline{\int_0^1} f \leq \frac{1}{2}$.
   (b) Show that the upper integral $\overline{\int_0^1} f = 1$.
   (c) Is f integrable? Explain.

2. Let $f : [a, b] \to \mathbb{R}$ be a continuous function such that $\int_c^d f \geq 0$ for all c, d with $a \leq c < d \leq b$. Prove that $f(x) \geq 0$ for all $x \in [a, b]$.

3. For numbers $a_0, a_1, \ldots, a_n$, define $p(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$ for all x. Suppose that
$$a_0 + \frac{a_1}{2} + \frac{a_2}{3} + \cdots + \frac{a_n}{n+1} = 0.$$
   Prove that there is an $x_0 \in (0, 1)$ such that $p(x_0) = 0$.

4. Let $f : [a, b] \to \mathbb{R}$ be monotonically increasing.
   (a) Show that f is bounded on $[a, b]$.
   (b) Let P_n be a regular partition of $[a, b]$ into n partition intervals. Show that
$$U(f, P_n) - L(f, P_n) = \frac{[f(b) - f(a)][b - a]}{n}$$