Math 142B
Midterm Exam 2
August 28, 2014
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Instructions

1. **You may use any type of calculator, but no other electronic devices during this exam.**
2. **You may use one page of notes, but no books or other assistance during this exam.**
3. **Write your *Name*, *PID*, and *Section* on the front of your Blue Book.**
4. **Write your solutions clearly in your Blue Book**
   (a) **Carefully indicate the number and letter of each question and question part.**
   (b) **Present your answers in the same order they appear in the exam.**
   (c) **Start each question on a new side of a page.**
5. **Read each question carefully, and answer each question completely.**
6. **Show all of your work; no credit will be given for unsupported answers.**

1. Given the polynomial $p(x) = x^4 - x + 1$, find constants c_0, c_1, c_2, c_3, c_4 so that $p(x) = c_0 + c_1(x-1) + c_2(x-1)^2 + c_3(x-1)^3 + c_4(x-1)^4$.

2. Recall that the hyperbolic cosine and hyperbolic sine functions are defined by

$$\cosh(x) = \frac{e^x + e^{-x}}{2}, \qquad \sinh(x) = \frac{e^x - e^{-x}}{2}.$$

and that they satisfy the identities

$$\cosh(0) = 1, \quad \sinh(0) = 0, \quad \cosh'(x) = \sinh(x), \quad \sinh'(x) = \cosh(x).$$

   (a) Verify that $p_n(x) = \sum_{k=0}^{n} \frac{1}{(2k)!} x^{2k} = 1 + \frac{1}{2}x^2 + \frac{1}{24}x^4 + \cdots + \frac{1}{(2n)!}x^{2n}$ is the n^{th} Taylor polynomial for $\cosh(x)$ centered at $x_0 = 0$. (Hint: Verify that $\cosh^{(2k)}(0) = 1$ and $\cosh^{(2k+1)}(0) = 0$ for each $k = 0, 1, 2, 3, \ldots$.)

   (b) Show that $\cosh(x) = \sum_{k=0}^{\infty} \frac{1}{(2k)!} x^{2k}$ for all x. That is, show that the Taylor series expansion for $\cosh(x)$ centered at $x_0 = 0$ is valid for all x.

Exam continues on other side.

3. The algebraic identity $\dfrac{1}{1+t^2} = \displaystyle\sum_{k=0}^{n}(-1)^k t^{2k} + \dfrac{(-1)^{n+1}t^{2n+2}}{1+t^2}$ implies that for every $n \in \mathbb{N}$ and every x, $\tan^{-1}(x) = \displaystyle\sum_{k=0}^{n} \dfrac{(-1)^k}{2k+1} x^{2k+1} + (-1)^{n+1} \int_0^x \dfrac{t^{2n+2}}{1+t^2}\, dt.$

$$\text{Prove that} \qquad \tan^{-1}(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} x^{2k+1} \quad \text{for} \quad |x| < 1.$$

4. Define $f_n(x) : (0,1) \to \mathbb{R}$ by

$$f_n(x) = \frac{1}{1-x} + \frac{(1-x^2)}{1-x} + \frac{(1-x^2)^2}{1-x} + \cdots + \frac{(1-x^2)^n}{1-x}.$$

Show that the sequence $\{f_n\}$ converges pointwise and determine the limit function $f : (0,1) \to \mathbb{R}$.