Math 120A
September 3, 2019

Question 1 $f(z)$ is analytic at ∞ if

A. there is $\rho > 0$ for which $f(z) = \sum_{k=0}^{\infty} \frac{b_k}{z^k}$ for all $|z| > \rho$.

B. $g(w) = f\left(\frac{1}{w}\right)$ is analytic at $w = 0$.

C. there is $\sigma > 0$ for which $f(z) = \sum_{k=0}^{\infty} a_k z^k$ for all $|z| < \sigma$.

*D. **A** and **B**.

E. **B** and **C**.

Question 2 Given $f(z) = \sum_{k=-\infty}^{\infty} b_k z^k$ for all $|z| > R$. $f(z)$ has a *removable* singularity at ∞ if

*A. $b_k = 0$ for all integers $k > 0$.

B. the principle part of $f(z)$ vanishes at ∞.

C. there is an integer $N \geq 1$ for which $b_N \neq 0$ but $b_k = 0$ for all integers $k > N$.

D. $b_k \neq 0$ for infinitely many integers $k > 0$.

E. none of the above; you can't remove singularities, especially at ∞.

Question 3 Given $f(z) = \sum_{k=-\infty}^{\infty} b_k z^k$ for all $|z| > R$. $f(z)$ has an *essential* singularity at ∞ if

A. $b_k = 0$ for all integers $k > 0$.

B. the principle part of $f(z)$ vanishes at ∞.

C. there is an integer $N \geq 1$ for which $b_N \neq 0$ but $b_k = 0$ for all integers $k > N$.

*D. $b_k \neq 0$ for infinitely many integers $k > 0$.

E. none of the above; singularities aren't essential, you can get by perfectly well without them.

Question 4 Given $f(z) = \sum_{k=-\infty}^{\infty} b_k z^k$ for all $|z| > R$. $f(z)$ has a *pole of order* N at ∞ if

A. $b_k = 0$ for all integers $k > 0$.

B. the principle part of $f(z)$ vanishes at ∞.

*C. there is an integer $N \geq 1$ for which $b_N \neq 0$ but $b_k = 0$ for all integers $k > N$.

D. $b_k \neq 0$ for infinitely many integers $k > 0$.

E. none of the above; poles are simple objects and don't need to be ordered by N.

Question 5 Given that $f(z)$ has a pole of order N at z_0. Then,

A. $f^{(k)}(z_0) = 0$ for integers k, $0 \leq k < N$ and $f^{(N)}(z_0) \neq 0$.

B. $h(z) = (z - z_0)^N f(z)$ is analytic at z_0 and $h(z_0) \neq 0$.

C. $f(z)$ has a Laurent series $\displaystyle\sum_{m=1}^{N} \frac{b_m}{(z - z_0)^m} + \sum_{n=0}^{\infty} a_n (z - z_0)^n$.

D. **A** and **B**.

*E. **B** and **C**.