Math 120A
September 5, 2019

Question 1 An *entire* function is a function that is analytic on the entire complex plane.
When $f(z)$ is a bounded entire function with $|f(z)| \leq M$ for all $z \in \mathbb{C}$, Cauchy's integral formula asserts that for every $z \in \mathbb{C}$ and every $R > 0$,

A. $f'(z) = \dfrac{1}{2\pi i} \displaystyle\int_{|\zeta - z| = R} \dfrac{f(\zeta)}{(\zeta - z)^2}\, d\zeta.$

B. $|f'(z)| \leq \dfrac{1}{2\pi} \dfrac{M}{R^2} \cdot 2\pi R = \dfrac{M}{R}$, by an *ML*-estimate.

C. $f'(z) = 0$ for every $z \in \mathbb{C}$.

D. **A** and **B**; **B** is an *ML*-estimate applied to **A**.

*E. all of the above; **C** follows by letting $R \to \infty$.

Question 2 Suppose $f(z)$ is complex-differentiable and $g(x, y)$ is real-differentiable. Which of the following properties does $g(x, y)$ share with $f(z)$?

A. $\oint_\gamma f(z)\, dz = 0$ along any simple closed path γ.

B. $f(z)$ has derivatives of all orders.

C. $f(z)$ is represented by a power series at every point.

D. all of the above; $\mathbb{R}^2$ is the same as $\mathbb{C}$.

*E. none of the above; $\mathbb{C}$ is not the same as $\mathbb{R}^2$.

Question 3 Why does $\log(z)$ have branches?

A. e^z is periodic.

B. You have to restrict the domain of e^z to obtain an invertible function.

C. There are many choices for a restricted domain on which e^z is invertible.

*D. All of the above.

E. None of the above.

Question 4 Suppose z_0 is a branch point of $f(z)$. Then,

*A. z_0 is a singularity of $f(z)$.

B. z_0 is an isolated singularity of $f(z)$.

C. $f(z)$ has a Laurent series expansion centered at z_0.

D. all of the above.

E. none of the above.

Question 5 In order to evaluate an integral of a function $f(z)$ with a branch point at z_0 using contour integration, you must

A. choose a contour that includes z_0 in the enclosed domain.

B. compute the residue of $f(z)$ at z_0 and apply the residue theorem.

*C. avoid the branch point by using a keyhole contour.

D. **A** and **B**

E. none of the above.

Question 6 Let $f(z) = e^z$ and $g(z) = z^{\frac{1}{4}}$.

A. $f(z)$ is single-valued, but $g(z)$ is multiple-valued.

B. $f\left(\frac{1}{4}\right) = g(e)$ since they are both equal to $e^{\frac{1}{4}}$.

C. $g(e) = \left\{e^{\frac{1}{4}+i\frac{\pi}{2}k},\ k = 0, 1, 2, 3\right\}$.

D. **B** and **C**

*E. **A** and **C**

Question 7 The power function z^α is single-valued

A. for every real number α.

B. for every rational number α.

*C. for every integer α.

D. All of the above; after all, every rational number is a real number and every integer is a rational number.

E. None of the above; z^α is always multiple-valued.

Question 8 Let n be a positive integer with $n \geq 2$, and let z be a nonzero complex number. Then,

*A. $z^{\frac{1}{n}}$ has n distinct values.

B. $z^{\frac{1}{n}}$ is single-valued.

C. $z^{\frac{1}{n}} \cdot z^{-\frac{1}{n}} = 1$.

D. **A** and **C**

E. **B** and **C**

Question 9 $f(z) = \dfrac{a\,z+b}{c\,z+d}$ is a fractional linear transformation.

We can conclude that f is defined at every point on

A. the complex plane $\mathbb{C}$.

B. the extended complex plane $\mathbb{C}^*$.

C. the Riemann sphere via stereographic projection.

D. **A** and **B**

*E. **B** and **C**

Question 10 Let $\gamma : [a, b] \to \mathbb{C}$ be a piecewise smooth path with length L. We can conclude

A. $\left|\int_\gamma dz\right| \leq L.$

B. $\int_\gamma |dz| = L.$

C. $\int_a^b |\gamma'(t)|\, dt = L.$

D. **B** and **C**; they are the same.

*E. all of the above.

Question 12 The functions $f_k : [0,1] \to \mathbb{R}$ given by $f_k(x) = x^k$

A. are all continous.

B. converge pointwise to a discontinuous function.

C. converge uniformly to a discontinous function.

*D. **A** and **B**.

E. all of the above; if they converge uniformly, they also converge pointwise.

Question 13 Suppose $\sum_{k=0}^{\infty} a_k z_0^k$ converges. We conclude that $\sum_{k=0}^{\infty} a_k z^k$

A. converges absolutely for every z with $|z| < |z_0|$.

B. converges uniformly for every z with $|z| \leq r$ whenever $r < |z_0|$.

C. converges absolutely for every z with $|z| = |z_0|$.

*D. **A** and **B**.

E. all of the above.

Question 14 Suppose $\sum_{k=0}^{\infty} a_k(z-(1+i))^2$ is the power series for $f(z) = \dfrac{1}{1+z^2}$ centered at $1+i$. It's radius of convergence is

*A. 1.

B. $\sqrt{5}$.

C. $R = \infty$ since $f(z)$ is defined for all z.

D. $R = 0$ since the power series converges only at $1+i$.

E. None of the above.

Question 15 The function $f(z) = \frac{1}{z} + \frac{1}{z^5} = \frac{z^4+1}{z^5}$. We can conclude

A. $f(z)$ has four simple zeros: $z \in \left\{ e^{i\frac{\pi}{4}}, e^{i\frac{3\pi}{4}}, e^{i\frac{5\pi}{4}}, e^{i\frac{7\pi}{4}} \right\}$.

B. $f(z)$ has a zero of order 5 at ∞.

C. $\frac{1}{z} + \frac{1}{z^5}$ is the Laurent series of f for $|z| > 0$.

D. **A** and **B**.

*E. **A** and **C**.