Math 120A
August 12, 2019

Question 1 Let $f(z)=e^{z}$ and $g(z)=z^{\frac{1}{4}}$.
A. $f(z)$ is single-valued, but $g(z)$ is multiple-valued.
B. $f\left(\frac{1}{4}\right)=g(e)$ since they are both equal to $e^{\frac{1}{4}}$.
C. $g(e)=\left\{e^{\frac{1}{4}+i \frac{\pi}{2} k}, k=0,1,2,3\right\}$.
D. B and C
*E. A and C

Question 2 A function $f(x, y)=(u(x, y), v(x, y))$ is complex differentiable at $z_{0}=\left(x_{0}, y_{0}\right)$ if and only if

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\begin{aligned}
& \text { A. } \frac{\partial u}{\partial x}=\frac{\partial v}{\partial y} \text { and } \frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x} \text { at }\left(x_{0}, y_{0}\right) \text {. } \\
& \text { B. } \frac{\partial}{\partial x}(u+i v)=\frac{1}{i} \frac{\partial}{\partial y}(u+i v) \text { at }\left(x_{0}, y_{0}\right) \text {. } \\
& \text { C. } \lim _{\Delta z \rightarrow 0} \frac{f\left(z_{0}+\Delta z\right)-f(z)}{\Delta z} \text { converges. } \\
& \text { D. } \mathbf{A} \text { and } \mathbf{C} \text {. } \\
& \text { *E. A, B, and C. }
\end{aligned}
$$

Note: B follows from A.

Question 3 The power function $z^{\alpha}$ is single-valued
A. for every real number $\alpha$.
B. for every rational number $\alpha$.
${ }^{*} C$. for every integer $\alpha$.
D. All of the above; after all, every rational number is a real number and every integer is a rational number.
E. None of the above; $z^{\alpha}$ is always multiple-valued.

