Math 120A
August 14, 2019

Question 1 $f(z) = \dfrac{a\,z + b}{c\,z + d}$ is a fractional linear transformation.

We can conclude that f is defined at every point on

A. the complex plane $\mathbb{C}$.

B. the extended complex plane $\mathbb{C}^*$.

C. the Riemann sphere via stereographic projection.

D. **A** and **B**

*E. **B** and **C**

Question 2 Let $f(z) = \dfrac{z+i}{z-i}$. Then,

A. $f(0) = -1$.

B. $f(1) = i$.

C. $f(\infty) = 1$.

*D. All of the above.

E. **A** and **B**. $f(\infty)$ is not defined because ∞ is not a complex number.

Question 3 Let $f(z) = e^z$ and $g(z) = z^{\frac{1}{4}}$.

A. $f(z)$ is single-valued, but $g(z)$ is multiple-valued.

B. $f\left(\frac{1}{4}\right) = g(e)$ since they are both equal to $e^{\frac{1}{4}}$.

C. $g(e) = \left\{e^{\frac{1}{4}+i\frac{\pi}{2}k},\ k = 0, 1, 2, 3\right\}$.

D. **B** and **C**

*E. **A** and **C**

Question 4 $f : \mathbb{R}^2 \to \mathbb{R}^2$ has derivative $\mathrm{Df} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$.

We can conclude that

A. f is complex differentiable because the partial derivatives of f satisfy the Cauchy-Riemann equations.

B. f is differentiable but not complex differentiable because $f'(z)$ cannot be written as a matrix.

C. f is complex differentiable and $|f'(z)|^2 = \det(\mathrm{Df}) = 2$.

*D. **A** and **C**

E. We can't conclude anything. Not enough information has been provided.

Question 5 Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ given by $f(x, y) = (u(x, y), v(x, y))$. Suppose f has continuous partial derivatives. Then,

A. f is differentiable.

B. $Df = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix}$ is the derivative of f.

C. Viewing f as $f(x + iy) = u(x + iy) + iv(x + iy)$, f is complex differentiable.

D. All of the above.

*E. **A** and **B**