Math 120A
August 19, 2019

Question 1 Recall that the differential $-\dfrac{y}{x^2+y^2}dx + \dfrac{x}{x^2+y^2}dy$ has the following two properties:

1. $\dfrac{\partial}{\partial y}\left(-\dfrac{y}{x^2+y^2}\right) = \dfrac{\partial}{\partial x}\left(\dfrac{x}{x^2+y^2}\right)$.
2. $\displaystyle\oint_{x^2+y^2=1} -\frac{y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy = 2\pi$.

Therefore, we can conclude that $-\dfrac{y}{x^2+y^2}dx + \dfrac{x}{x^2+y^2}dy$

*A. is closed.

B. is exact.

C. is both closed and exact.

D. is neither closed nor exact.

E. violates Green's theorem.

Question 2 Consider the functions $u(x, y) = y^2 - x^2$ and $v(x, y) = -2xy$. We can conclude

A. $u(x, y)$ and $v(x, y)$ are harmonic.

B. $u(x, y)$ and $v(x, y)$ are harmonic conjugates.

C. $f(x, y) = u(x, y) + iv(x, y)$ is analytic.

D. **A** and **B**.

*E. All of the above.

Question 3 Define $\log_k(z) := \mathrm{Log}(z) + i\,2\pi k$. Then,

A. $\mathrm{Log}'(z) = \frac{1}{z}$.

B. $\log_k'(z) = \frac{1}{z}$ for every integer k.

C. **A** and **B**.

*D. all of the above, provided $z \in \mathbb{C} \setminus (-\infty, 0]$.

E. none of the above.