Math 120A
August 27, 2019

Question 1 Let $f(z) = \dfrac{1}{1+z^2}$.

A. $f(z) = \sum_{k=0}^{\infty} (-1)^k z^{2k}$ is the power series for f centered at 0 and converges for $|z| < 1$.

B. $f(z) = \sum_{k=1}^{\infty} \dfrac{(-1)^{k+1}}{z^{2k}}$ is the power series for f centered at ∞ and converges for $|z| > 1$.

C. $f(z)$ has a zero at ∞.

D. **A** and **C**.

*E. All of the above.

Question 2 Suppose $\sum_{k=0}^{\infty} a_k(z-(1+i))^2$ is the power series for $f(z) = \dfrac{1}{1+z^2}$ centered at $1+i$. It's radius of convergence is

*A. 1.

B. $\sqrt{5}$.

C. $R = \infty$ since $f(z)$ is defined for all z.

D. $R = 0$ since the power series converges only at $1+i$.

E. None of the above.

Question 3 Suppose $\sum_{k=0}^{\infty} a_k z_0^k$ converges. We conclude that $\sum_{k=0}^{\infty} a_k z^k$

A. converges absolutely for every z with $|z| < |z_0|$.

B. converges uniformly for every z with $|z| \leq r$ whenever $r < |z_0|$.

C. converges absolutely for every z with $|z| = |z_0|$.

*D. **A** and **B**.

E. all of the above.

Question 4 Let γ be the curve $|z| = 2$ with positive (counterclockwise) orientation. Then the integral $\displaystyle\int_\gamma \frac{z^n}{z-1}\,dz$

A. is equal to 0 by Cauchy's theorem.

B. is equal to 1 by the Cauchy integral theorem.

*C. is equal to $2\pi i$ by the Cauchy integral theorem.

D. is undefined because $\dfrac{z^n}{z-1}$ is undefined at $z = 1$.

E. none of the above.

Question 5 Let γ be the curve $|z| = 2$ with positive (counterclockwise) orientation. Then the integral $\int_\gamma \frac{z^n}{z+2}\, dz$

A. is equal to 0 by Cauchy's theorem.

B. is equal to $(-2)^n$ by the Cauchy integral theorem.

C. is equal to $2\pi i\,(-2)^2$ by the Cauchy integral theorem.

*D. is undefined because $\frac{z^n}{z+2}$ is undefined at $z = -2$.

E. none of the above.