Math 120A
August 28, 2019

Question 1 The function $f(z) = \frac{1}{z^2 - z} = -\frac{1}{z} \cdot \frac{1}{1-z}$ can be decomposed as

A. $f(z) = f_0(z) + f_1(z)$ with $f_0(z)$ analytic for $|z| < 1$ and $f_1(z)$ analytic for $|z| > 0$ and vanishes at ∞.

B. $f(z) = -\frac{1}{z} - \sum_{k=0}^{\infty} z^k$.

C. **A** and **B**; $f_0(z) = -\sum_{k=0}^{\infty} z^k$ and $f_1(z) = -\frac{1}{z}$.

*D. All of the above; this is an example of a Laurent decomposition analytic for $0 < |z| < 1$.

E. None of the above. I don't even know who Laurent was ...

Question 2 We can also write $f(z) = \frac{1}{z^2 - z} = \frac{1}{z^2} \cdot \frac{1}{1 - \left(\frac{1}{z}\right)}$. Thus,

A. $f(z) = \sum_{k=2}^{\infty} \frac{1}{z^k}$ for $|z| > 1$.

B. $f(z) = f_0(z) + f_1(z)$, where $f_0(z)$ is analytic for $|z| < +\infty$ and $f_1(z)$ is analytic for $|z| > 1$ and vanishes at ∞.

C. **A** and **B** and $f_0(z)$ is identically zero.

*D. All of the above; this is an example of a Laurent decomposition analytic for $|z| > 1$.

E. None of the above. Did Laurent ever meet Cauchy?

Question 3 Let A be the annulus $\rho < |z - z_0| < \sigma$ with boundary $\partial A = \{|z - z_0| = \sigma\} \cup \{|z - z_0| = \rho\}$. If $f(z)$ is analytic on A and extends smoothly to ∂A, then for all $z \in A$:

A. $f(z) = \frac{1}{2\pi i} \int_{\partial A} \frac{f(\zeta)}{\zeta - z}\, d\zeta$, traversing ∂A along its positive orientation.

B. The positive orientation of ∂A is *counterclockwise* along the outer circle $\{|z - z_0| = \sigma\}$ and *clockwise* along the inner circle $\{|z - z_0| = \rho\}$

C. $f(z) = \frac{1}{2\pi i} \int_{|z-z_0|=\sigma} \frac{f(\zeta)}{\zeta - z}\, d\zeta - \frac{1}{2\pi i} \int_{|z-z_0|=\rho} \frac{f(\zeta)}{\zeta - z}\, d\zeta$, traversing both circles in *counterclockwise* orientation.

*D. All of the above; the result is called Cauchy's integral formula for an annulus.

E. None of the above; not much can be done on an annulus.

Question 4 The function $f(z) = \frac{1}{z} + \frac{1}{z^5} = \frac{z^4+1}{z^5}$. We can conclude

A. $f(z)$ has four simple zeros: $z \in \left\{ e^{i\frac{\pi}{4}}, e^{i\frac{3\pi}{4}}, e^{i\frac{5\pi}{4}}, e^{i\frac{7\pi}{4}} \right\}$.

B. $f(z)$ has a zero of order 5 at ∞.

C. $\frac{1}{z} + \frac{1}{z^5}$ is the Laurent series of f for $|z| > 0$.

D. **A** and **B**.

*E. **A** and **C**.

Question 5 Suppose $\sum_{k=0}^{\infty} a_k(z-(1+i))^2$ is the power series for $f(z) = \frac{1}{1+z^2}$ centered at $1+i$. It's radius of convergence is

*A. 1.

B. $\sqrt{5}$.

C. $R = \infty$ since $f(z)$ is defined for all z.

D. $R = 0$ since the power series converges only at $1+i$.

E. None of the above.