Math 120A
Midterm Exam 1
August 15, 2013
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Instructions

1. You may use any type of calculator, but no other electronic devices during this exam.
2. You may use one page of notes, but no books or other assistance during this exam.
3. Write your *Name*, *PID*, and *Section* on the front of your Blue Book.
4. Write your solutions clearly in your Blue Book

    (a) Carefully indicate the number and letter of each question and question part.
    (b) Present your answers in the same order they appear in the exam.
    (c) Start each question on a new side of a page.

5. Read each question carefully, and answer each question completely.
6. Show all of your work; no credit will be given for unsupported answers.

1. (6 points) Given that $a + ib = \dfrac{x - i\,y}{x + i\,y}$, show that $a^2 + b^2 = 1$.

2. (6 points) Find the cube roots of i and write them in the form $a + i\,b$.

3. (6 points) Let $S = \{z \mid z = x + i\}$, the horizontal line in $\mathbb{C}$ with $\operatorname{Im}(z) = 1$.
   Determine the image $S' = f(S)$ under the mapping $f(z) = \dfrac{i}{z}$.

4. (6 points) Let $f(z) = |z|^2$.

    (a) Show that f is differentiable at $z = 0$ by evaluating $f'(0)$.
    (b) Show that f is *not* differentiable at any point $z \neq 0$.

5. (6 points) Suppose $f(z) = u(x, y) + i\,v(x, y)$ is analytic.

    (a) Show that the function $\phi(x, y) = u(x, y)\,v(x, y)$ is harmonic.
    (b) Find $\psi(x, y)$ so that $g(z) = \phi(x, y) + i\,\psi(x, y)$ is analytic.