Math 142B **Midterm Exam 1 v. A** **Name:** ____________

August 16, 2018 **(Total Points: *25*)** **PID:** ____________

Instructions

1. **Write your Name and PID in the spaces provided above.**
2. **Make sure your Name is on every page.**
3. **No calculators, tablets, phones, or other electronic devices are allowed during this exam.**
4. **Put away ANY devices that can be used for communication or can access the Internet.**
5. **You may use one handwritten page of notes, but no books or other assistance during this exam.**
6. **Read each question carefully and answer each question completely.**
7. **Write your solutions clearly in the spaces provided.**
8. **Show all of your work. No credit will be given for unsupported answers, even if correct.**

(1 point) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(6 points) 1. Exhibit an example of each of the following functions. Be sure to include a brief explanation for why the function you chose is an example with the required properties.

(a) A function $f : [a, b] \to \mathbb{R}$ that is not bounded above.

(b) A bounded function $f : [a, b] \to \mathbb{R}$ that has no minimum.

(6 points) 2. Let $f_1 : (-1, 1) \to \mathbb{R}$ be defined by

$$f_1(x) = \frac{1}{1 - x^2},$$

and $f_2 : (-1, 1) \to \mathbb{R}$ be defined by

$$f_2(x) = \sin\left(\frac{1}{1 - x^2}\right).$$

(a) Extend f_1 to $f_1 : [-1, 1] \to \mathbb{R}$ by defining $f_1(-1) = f_1(1) = 0$. Is f_1 integrable on $[-1, 1]$? Explain.

(b) Extend f_2 to $f_2 : [-1, 1] \to \mathbb{R}$ by defining $f_2(-1) = f_2(1) = 0$. Is f_2 integrable on $[-1, 1]$? Explain.

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(6 points) 3. Let $f : [a, b] \to \mathbb{R}$ be a continuous function such that $\int_c^d f \geq 0$ for all $[c, d] \subseteq [a, b]$. Prove that $f(x) \geq 0$ for all $x \in [a, b]$.

(6 points) 4. Let $f : [a, b] \to \mathbb{R}$ be monotonically increasing.

(a) Show that f is bounded on $[a, b]$.

(b) Let P_n be the regular partition of $[a, b]$ into n partition intervals. Show that

$$\lim_{n\to\infty} U(f, P_n) - L(f, P_n) = 0.$$