Math 102 Homework Assignment 1
Due Thursday, January 13, 2022

1. The inverse of $\begin{bmatrix} I & 0 & 0 \\ C & I & 0 \\ A & B & I \end{bmatrix}$ is $\begin{bmatrix} I & 0 & 0 \\ Z & I & 0 \\ X & Y & I \end{bmatrix}$.

   Find X, Y, and Z.

2. Let $A = \begin{bmatrix} A_{11} & A_{12} \\ O & A_{22} \end{bmatrix}$, with all four blocks are $n \times n$ matrices and A_{11} and A_{22} nonsingular.

   (a) Show that A is nonsingular and that A^{-1} is of the form $\begin{bmatrix} A_{11}^{-1} & C \\ O & A_{22}^{-1} \end{bmatrix}$.

   (b) Determine C.

3. Let $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$, $\mathbf{u}_3 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$.

   (a) Find the transition matrix corresponding to the change of basis from $\mathcal{E} = [\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3]$ to $\mathcal{U} = [\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3]$.

   (b) Find the coordinates of each of the following vectors with respect to the basis $\mathcal{U}$:

   (i) $\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ (ii) $\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$

4. (a) Find the transition matrix representing the change of coordinates on P_3 for the ordered basis $\mathcal{E} = \left[1,\ x,\ x^2\right]$ to the ordered basis $\mathcal{D} = \left[1,\ 1+x,\ 1+x+x^2\right]$.

   (b) Find the coordinates $[p]_{\mathcal{D}}$ for the polynomial $p = 3 + 2x + x^2$ with respect to the ordered basis $\mathcal{D}$.

5. Let L be a linear operator on $\mathbb{R}^1$ such that $L(1) = a$. Show that $L(x) = ax$ for every $x \in \mathbb{R}^1$.

6. Let $[\mathbf{v}_1, \ldots, \mathbf{v}_n]$ be a basis for a vector space V, and let L_1 and L_2 be two linear transformations mapping V into a vector space W. Show that if $L_1(\mathbf{v}_i) = L_2(\mathbf{v}_i)$ for each $i = 1, \ldots, n$, then $L_1 = L_2$; that is, $L_1(\mathbf{v}) = L_2(\mathbf{v})$ for *every* $\mathbf{v} \in V$.