1. **Exercise A1:** Using the orthonormal basis (found in Exercise 5.6.4) for the subspace spanned by \( \{1, x, x^2\} \) in the vector space of polynomials with inner product \( \langle f, g \rangle = \int_{-1}^{1} f(t)g(t) \, dt \), find the quadratic polynomial that best approximates \( f(x) = \cos(x) \) on \([-1, 1]\).

2. **Exercise A2:** Find a QR factorization for the matrix
\[
A = \begin{pmatrix}
-2 & 3 \\ 5 & 7 \\ 2 & -2 \\ 4 & 6
\end{pmatrix}.
\]

3. **Exercise A3:** Suppose that \( A = QR \), where \( Q \) is a \( m \times n \) matrix and \( R \) is a \( n \times n \) matrix. Show that if the columns of \( A \) are linearly independent, then \( R \) is invertible. (Note: \( Q \) is not assumed to be an orthogonal matrix.)

4. **Exercise A4:** Suppose that \( A = QR \), where \( R \) is an invertible matrix. Show that \( A \) and \( Q \) have the same column space.

5. **Exercise A5:** Given \( \{u_1, \ldots, u_p\} \) an orthogonal basis for a subspace \( W \) of \( \mathbb{R}^n \).

   Let \( T : \mathbb{R}^n \to \mathbb{R}^n \) be defined by \( T(x) = \text{proj}_W x \), the projection of \( x \) onto the subspace \( W \).

   (a) Verify that \( T \) is a linear transformation.

   (b) What is \( \ker(T) \), the kernel of \( T \)?

   (c) What is \( T(\mathbb{R}^n) \), the range of \( T \)?