Question 1  \( f(z) \) is analytic at \( \infty \) if

A. there is \( \rho > 0 \) for which \( f(z) = \sum_{k=0}^{\infty} \frac{b_k}{z^k} \) for all \( |z| > \rho \).

B. \( g(w) = f \left( \frac{1}{w} \right) \) is analytic at \( w = 0 \).

C. there is \( \sigma > 0 \) for which \( f(z) = \sum_{k=0}^{\infty} a_k z^k \) for all \( |z| < \sigma \).

*D. A and B.*

E. B and C.
Question 2  Given \( f(z) = \sum_{k=-\infty}^{\infty} b_k z^k \) for all \( |z| > R \). \( f(z) \) has a removable singularity at \( \infty \) if

A. \( b_k = 0 \) for all integers \( k > 0 \).

B. the principle part of \( f(z) \) vanishes at \( \infty \).

C. there is an integer \( N \geq 1 \) for which \( b_N \neq 0 \) but \( b_k = 0 \) for all integers \( k > N \).

D. \( b_k \neq 0 \) for infinitely many integers \( k > 0 \).

E. none of the above; you can’t remove singularities, especially at \( \infty \).
Question 3  Given \( f(z) = \sum_{k=-\infty}^{\infty} b_k z^k \) for all \( |z| > R \). \( f(z) \) has an essential singularity at \( \infty \) if

A. \( b_k = 0 \) for all integers \( k > 0 \).

B. the principle part of \( f(z) \) vanishes at \( \infty \).

C. there is an integer \( N \geq 1 \) for which \( b_N \neq 0 \) but \( b_k = 0 \) for all integers \( k > N \).

*D. \( b_k \neq 0 \) for infinitely many integers \( k > 0 \).

E. none of the above; singularities aren’t essential, you can get by perfectly well without them.
Question 4  Given $f(z) = \sum_{k=-\infty}^{\infty} b_k z^k$ for all $|z| > R$. $f(z)$ has a pole of order $N$ at $\infty$ if

A. $b_k = 0$ for all integers $k > 0$.

B. the principle part of $f(z)$ vanishes at $\infty$.

*C. there is an integer $N \geq 1$ for which $b_N \neq 0$ but $b_k = 0$ for all integers $k > N$.

D. $b_k \neq 0$ for infinitely many integers $k > 0$.

E. none of the above; poles are simple objects and don’t need to be ordered by $N$. 
Question 5  Given that \( f(z) \) has a pole of order \( N \) at \( z_0 \). Then,

A. \( f^{(k)}(z_0) = 0 \) for integers \( k, 0 \leq k < N \) and \( f^{(N)}(z_0) \neq 0 \).

B. \( h(z) = (z - z_0)^N f(z) \) is analytic at \( z_0 \) and \( h(z_0) \neq 0 \).

C. \( f(z) \) has a Laurent series \( \sum_{m=1}^{N} \frac{b_m}{(z - z_0)^m} + \sum_{n=0}^{\infty} a_n(z - z_0)^n \).

D. A and B.

*E. B and C.