Question 1  Given \( f(z) = \sum_{k=-\infty}^{\infty} b_k z^k \) for all \(|z| > R\). \( f(z) \) has a removable singularity at \( \infty \) if

*A. \( b_k = 0 \) for all integers \( k > 0 \).

B. the principle part of \( f(z) \) vanishes at \( \infty \).

C. there is an integer \( N \geq 1 \) for which \( b_N \neq 0 \) but \( b_k = 0 \) for all integers \( k > N \).

D. \( b_k \neq 0 \) for infinitely many integers \( k > 0 \).

E. none of the above; you can’t remove singularities, especially at \( \infty \).
**Question 2**  A function $f(z)$ has a nonzero residue at $z_0$ if

A. $z_0$ is an isolated singularity of $f(z)$

B. the principal part of $f(z)$ is not zero.

*C. $z_0$ is the only singularity of $f(z)$ in $|z - z_0| < \rho$ and
\[
\int_{|\zeta - z_0| = \epsilon} f(\zeta) \, d\zeta \neq 0 \text{ for every } 0 < \epsilon < \rho.
\]

D. all of the above.

E. none of the above.
**Question 3**  Suppose $f(z)$ has an essential singularity at $z_0$. Then,

A. Res $[f(z), z_0]$, the residue of $f(z)$ at $z_0$, is undefined.

B. $\int_{|\zeta - z_0| = \epsilon} f(\zeta) \, d\zeta$ is not defined for any $\epsilon > 0$.

C. $f(z)$ is not analytic at $\infty$.

D. all of the above.

*E. none of the above.
Question 4  Let \( f(z) = \frac{1}{(z - z_0)^2} \). Then,

A. \( \int_{|\zeta - z_0| = \epsilon} f(\zeta) \, d\zeta = 0 \) for every \( \epsilon > 0 \).

B. \( \int_{|\zeta - z_0| = \epsilon} f(\zeta) \, d\zeta = 2\pi i \) for every \( \epsilon > 0 \).

C. \( \text{Res}[f(z), z_0] = 0 \).

* D. A and C.

E. none of the above.
Question 5 Let \( f(z) = \frac{1}{(z - z_0)} \). Then,

A. \( \int_{|\zeta - z_0| = \epsilon} f(\zeta) \, d\zeta = 0 \) for every \( \epsilon > 0 \).

*B. \( \int_{|\zeta - z_0| = \epsilon} f(\zeta) \, d\zeta = 2\pi i \) for every \( \epsilon > 0 \).

C. \( \text{Res}[f(z), z_0] = 0 \).

D. A and C.

E. none of the above.