**Question 1**  The function \( f(z) = \frac{1}{z^2 - z} = -\frac{1}{z} \cdot \frac{1}{1-z} \) can be decomposed as

A. \( f(z) = f_0(z) + f_1(z) \) with \( f_0(z) \) analytic for \( |z| < 1 \) and \( f_1(z) \) analytic for \( |z| > 0 \) and vanishes at \( \infty \).

B. \( f(z) = -\frac{1}{z} - \sum_{k=0}^{\infty} z^k \).

C. A and B; \( f_0(z) = -\sum_{k=0}^{\infty} z^k \) and \( f_1(z) = -\frac{1}{z} \).

*D. All of the above; this is an example of a Laurent decomposition analytic for \( 0 < |z| < 1 \).*

E. None of the above. I don’t even know who Laurent was ...
Question 2  We can also write $f(z) = \frac{1}{z^2 - z} = \frac{1}{z^2} \cdot \frac{1}{1 - \left(\frac{1}{z}\right)}$. Thus,

A. $f(z) = \sum_{k=2}^{\infty} \frac{1}{z^k}$ for $|z| > 1$.

B. $f(z) = f_0(z) + f_1(z)$, where $f_0(z)$ is analytic for $|z| < +\infty$ and $f_1(z)$ is analytic for $|z| > 1$ and vanishes at $\infty$.

C. A and B and $f_0(z)$ is identically zero.

*D. All of the above; this is an example of a Laurent decomposition analytic for $|z| > 1$.

E. None of the above. Did Laurent ever meet Cauchy?
**Question 3** Let \( A \) be the annulus \( \rho < |z - z_0| < \sigma \) with boundary \( \partial A = \{|z - z_0| = \sigma\} \cup \{|z - z_0| = \rho\} \). If \( f(z) \) is analytic on \( A \) and extends smoothly to \( \partial A \), then for all \( z \in A \):

A. \( f(z) = \frac{1}{2\pi i} \int_{\partial A} \frac{f(\zeta)}{\zeta - z} \, d\zeta \), traversing \( \partial A \) along its positive orientation.

B. The positive orientation of \( \partial A \) is *counterclockwise* along the outer circle \( \{|z - z_0| = \sigma\} \) and *clockwise* along the inner circle \( \{|z - z_0| = \rho\} \)

C. \( f(z) = \frac{1}{2\pi i} \int_{|z-z_0|=\sigma} \frac{f(\zeta)}{\zeta - z} \, d\zeta - \frac{1}{2\pi i} \int_{|z-z_0|=\rho} \frac{f(\zeta)}{\zeta - z} \, d\zeta \), traversing both circles in *counterclockwise* orientation.

*D. All of the above; the result is called Cauchy’s integral formula for an annulus.*

E. None of the above; not much can be done on an annulus.
The function $f(z) = \frac{1}{z} + \frac{1}{z^5} = \frac{z^4 + 1}{z^5}$. We can conclude

A. $f(z)$ has four simple zeros: $z \in \left\{ e^{i \frac{\pi}{4}}, e^{i \frac{3\pi}{4}}, e^{i \frac{5\pi}{4}}, e^{i \frac{7\pi}{4}} \right\}$.

B. $f(z)$ has a zero of order 5 at $\infty$.

C. $\frac{1}{z} + \frac{1}{z^5}$ is the Laurent series of $f$ for $|z| > 0$.

D. A and B.

*E. A and C.
Question 5  Suppose \( \sum_{k=0}^{\infty} a_k(z - (1 + i))^2 \) is the power series for \( f(z) = \frac{1}{1 + z^2} \) centered at \( 1 + i \). It’s radius of convergence is

* A. 1.

B. \( \sqrt{5} \).

C. \( R = \infty \) since \( f(z) \) is defined for all \( z \).

D. \( R = 0 \) since the power series converges only at \( 1 + i \).

E. None of the above.