

Math 142A Homework Assignment 3
Due Wednesday, October 25

1. Let $\{a_n\}$ be a monotone sequence.
 - (a) Prove that $\{a_n\}$ converges if and only if $\{a_n^2\}$ converges.
 - (b) Show that the result in part (a) is false if $\{a_n\}$ is not monotone.
2. Let $\{a_n\}$ and $\{b_n\}$ be sequences such that $a_n < b_n$ for every index n . For each index n , define the interval $I_n = [a_n, b_n]$. Suppose that

$$I_{n+1} \subseteq I_n \quad \text{for every index } n.$$

Using the Monotone Convergence Theorem, prove that

- (a) $a_n \rightarrow a$ and $b_n \rightarrow b$ with $a \leq b$, and
 - (b) $[a, b] \subseteq I_n$ for every index n .
3. Show that a strictly increasing sequence has no peak indices.
4. Show that every index of a monotonically decreasing sequence is a peak index.
5. Suppose $\{a_n\}$ is a monotone sequence with a subsequence $\{a_{n_k}\}$ such that $a_{n_k} \rightarrow a$. Prove that $a_n \rightarrow a$.