**Question 1**  For each index $n$, let $f_n : [0, 1] \rightarrow \mathbb{R}$ be given by

$$f_n(x) = \begin{cases} 1 & \text{if } x = \frac{k}{2^n} \text{ for some integer } k, \ 0 \leq k \leq 2^n \\ 0 & \text{otherwise} \end{cases}$$

Let $f : [0, 1] \rightarrow \mathbb{R}$ be given by $f(x) = \lim_{n \to \infty} f_n(x)$ for each $x \in [0, 1]$.

Then,

- **A.** $\int_0^1 f_n = 0$ for every index $n$.
- **B.** $\int_0^1 f = 0$ and $\int_0^1 f = 1$.
- **C.** $\int_0^1 f = 0$.
- **D.** A and B.
- **E.** A and C.
Question 2  For each index \( n \), let \( f_n : [0, 1] \to \mathbb{R} \) be given by

\[
f_n(x) = \begin{cases} 
  n^2 x & \text{if } 0 \leq x < \frac{1}{n} \\
  2n - n^2 x & \text{if } \frac{1}{n} \leq x < \frac{2}{n} \\
  0 & \text{if } \frac{2}{n} \leq x \leq 1
\end{cases}
\]

Let \( f : [0, 1] \to \mathbb{R} \) be given by \( f(x) = \lim_{n \to \infty} f_n(x) \) for each \( x \in [0, 1] \).

Then,

A. \( \int_0^1 f_n = 1 \) for every index \( n \).

B. \( \int_0^1 f = 0 \).

C. \( \int_0^1 f = \lim_{n \to \infty} \int_0^1 f_n. \)

*D.  A and B.

E.  A and C.
Question 3  For each natural number $n$, define $f_n : [0, 1] \to \mathbb{R}$ by 
$$f_n(x) = \sum_{k=0}^{n} \frac{1}{k!} x^k.$$ Define $f : [0, 1] \to \mathbb{R}$ by $f(x) = e^x$. Then, 
$\{f_n\}$ converges pointwise on $[0, 1]$ to $f$. We can also say that

A. $\{f_n\}$ converges uniformly to $f$ on $[0, 1]$.

B. $\lim_{n \to \infty} \int_0^1 f_n = \int_0^1 f$.

C. $\lim_{n \to \infty} f'_n(x) = f'(x)$ for each $x \in (0, 1)$.

D. B and C.

*E. All of the above.
Question 4 Given a sequence of functions \( \{f_n : [a, b] \to \mathbb{R}\} \) such that \( \{f_n\} \) converges pointwise to \( f \) on \( [a, b] \). Then we can say that

A. if \( f_n \) is integrable for every index \( n \), then \( f \) is integrable.

B. if \( \int_a^b f_n = 1 \) for every index \( n \), then \( \int_a^b f = 1 \).

C. if \( f_n \) is continuous for every index \( n \), then \( f \) is continuous.

D. All of the above.

*E. None of the above.
**Question 5**  A sequence \( \{a_k\} \) is Cauchy (or, is a Cauchy sequence) if for every \( \varepsilon > 0 \), there is an index \( N \) such that \( |a_m - a_n| < \varepsilon \) for all indices \( m, n \geq N \).

Given that \( \{b_k\} \) is a Cauchy sequence. Then \( \{b_k\} \) is

A. convergent.
B. bounded.
C. sequentially compact.
*D. A and B.
E. B and C.
Question 6  A sequence of functions \( \{f_k\} \) is uniformly Cauchy on \( D \) if for every \( \varepsilon > 0 \), there is an index \( N \) such that \( |f_m(x) - f_n(x)| < \varepsilon \) for all indices \( m, n \geq N \) and all \( x \in D \).

Given that \( \{g_k\} \) is uniformly Cauchy on \([0, 1]\). Then,

A. For each \( x \in [0, 1] \), \( \{g_k(x)\} \) is a Cauchy sequence.
B. \( \{g_k\} \) converges pointwise to a function \( g : [0, 1] \rightarrow \mathbb{R} \).
C. \( \{g_k\} \) converges uniformly to a function \( g : [0, 1] \rightarrow \mathbb{R} \).
D. A and B.
*E. A, B, and C.