Question 1  Setting \( f_n(x) = \sum_{k=0}^{n} (-1)^k x^{2k} \) for \( x \in [0, 1] \)
defines a sequence \( \{f_n : [0, 1] \to \mathbb{R}\} \).

Let \( f : [0, 1] \to \mathbb{R} \) be defined by \( f(x) = \frac{1}{1+x^2} \). Then,

* A. \( \int_0^1 f = \lim_{n \to \infty} \int_0^1 f_n; \) this is the Newton-Gregory Formula.

 B. \( \{f_n\} \) converges pointwise on \([0, 1]\) to \( f \).

 C. \( \{f_n\} \) converges uniformly on \([0, 1]\) to \( f \).

 D. All of the above.

 E. None of the above.
Question 2  Given a series $\sum_{k=1}^{\infty} a_k$ such that $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = L$. Then,

A. $\sum_{k=1}^{\infty} a_k$ converges conditionally whenever $L = 1$.

B. $\sum_{k=1}^{\infty} a_k$ converges absolutely whenever $L < 1$.

C. $\sum_{k=1}^{\infty} a_k$ diverges whenever $L > 1$.

D. A and B.

*E. B and C.
Question 3  Setting \( f_n(x) = \frac{1}{n} \tan^{-1}(n^2x) \) defines a sequence \( \{f_n : \mathbb{R} \to \mathbb{R}\} \) of infinitely differentiable functions. In addition,

A. \( \{f_n\} \) converges pointwise on \( \mathbb{R} \) to 0 since
\[
\lim_{n \to \infty} \frac{1}{n} \tan^{-1}(n^2x) = 0 \quad \text{for all } x.
\]

B. \( \{f_n\} \) converges uniformly on \( \mathbb{R} \) to 0 since
\[
|f_n(x)| < \frac{\pi}{2n} \quad \text{for all } x.
\]

C. \( \{f'_n(0)\} \) is unbounded, even though \( \{f_n\} \) converges uniformly on \( \mathbb{R} \).

D. A and B.

E. A, B, and C.
**Question 4**  Given a power series \( \sum_{k=0}^{\infty} c_k x^k \) with \( D \) its domain of convergence. Let \( r > 0 \) such that \((-r, r) \subseteq D\) and define \( f : (-r, r) \to \mathbb{R} \) by \( f(x) = \sum_{k=0}^{\infty} c_k x^k \). Then,

A. \( f \) is infinitely differentiable.

B. \( \frac{d^n}{dx^n} f(x) = \sum_{k=0}^{\infty} c_k \frac{d^n}{dx^n} x^k \) for each index \( n \).

C. \( \frac{f^{(n)}(0)}{n!} = c_n \) for each index \( n \).

* D. All of the above.

E. None of the above.
Question 5  Given a power series \( \sum_{k=0}^{\infty} c_k x^k \) with a bounded domain of convergence \( D \). Define \( R := \sup D \). Then,

A. \((-R, R) \subseteq D \subseteq [-R, R]\).

B. \( D \) is one of: \((-R, R), [-R, R), (-R, R], \) or \([ -R, R] \).

C. \( R \) is called the radius of convergence of \( \sum_{k=0}^{\infty} c_k x^k \).

*D. All of the above.

E. None of the above.
Question 6  A sequence of functions \( \{f_k\} \) is uniformly Cauchy on \( D \) if for every \( \varepsilon > 0 \), there is an index \( N \) such that \( |f_m(x) - f_n(x)| < \varepsilon \) for all indices \( m, n \geq N \) and all \( x \in D \).

Given that \( \{g_k\} \) is uniformly Cauchy on \([0, 1]\). Then,

A. For each \( x \in [0, 1] \), \( \{g_k(x)\} \) is a Cauchy sequence.

B. \( \{g_k\} \) converges pointwise to a function \( g : [0, 1] \rightarrow \mathbb{R} \).

C. \( \{g_k\} \) converges uniformly to a function \( g : [0, 1] \rightarrow \mathbb{R} \).

D. A and B.

*E. A, B, and C.
Question 7  Clicker questions were used throughout this course to think about the ideas in new ways and to review previously considered ideas. I found the clicker questions to be

A. a helpful aid for study and review.
B. an amusing way to begin each class.
C. a complete waste of time; I would have rather slept longer.
D. each of the above, depending on which day it was.
E. none of the above; I think I’m getting tinnitus.