1. Let $A = \begin{bmatrix} 1 & -3 & -4 \\ -3 & 2 & 6 \\ 5 & -1 & -8 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$.
   (a) Show that the matrix equation $A\mathbf{x} = \mathbf{b}$ does not have a solution for all possible $\mathbf{b}$; that is, show that there exists at least one $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ does not have a solution.
   (b) Describe the set of all $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ *does* have a solution.

2. Let $\mathbf{v}_1 = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ -3 \\ 8 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 4 \\ -1 \\ -5 \end{bmatrix}$.

   Determine whether or not $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ spans $\mathbb{R}^3$. Clearly explain your reasoning.

3. Let $\mathbf{p} = \begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix}$ and $\mathbf{q} = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}$.

   Find a parametric equation for the line L passing through $\mathbf{p}$ and $\mathbf{q}$.

   (*Hint*: L is parallel to $\mathbf{q} - \mathbf{p}$.)

4. Suppose $A\mathbf{x} = \mathbf{b}$ has a solution. Explain why the solution is unique precisely when $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

5. Find the value(s) of h for which the following set of vectors is linearly *dependent*, and justify your answer.

$$\left\{ \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}, \begin{bmatrix} -6 \\ 7 \\ 3 \end{bmatrix}, \begin{bmatrix} 8 \\ h \\ 4 \end{bmatrix} \right\}$$

6. Suppose A is a $m \times n$ matrix with the property that for all $\mathbf{b} \in \mathbb{R}^m$ the equation $A\mathbf{x} = \mathbf{b}$ has at most one solution. Use the definition of linear independence to explain why the columns of A must be linearly independent.