Math 18 **Homework 3** **Due January 27, 2020**

1. Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = mx + b$.
   (a) Show that f is a linear transformation when $b = 0$.
   (b) Find a property of linear transformations that is violated when $b \neq 0$.

2. Let $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, and let $\mathbf{y}_1 = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$, $\mathbf{y}_2 = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}$, $\mathbf{y}_3 = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$.

   Suppose $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation that maps $\mathbf{e}_1$ to $\mathbf{y}_1$, $\mathbf{e}_2$ to $\mathbf{y}_2$, and $\mathbf{e}_3$ to $\mathbf{y}_3$.
   (a) Find the image of $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ under T.
   (b) Is T one-to-one? Justify your answer.
   (c) Does T map $\mathbb{R}^3$ onto $\mathbb{R}^3$? Justify your answer.

3. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first reflects points through the horizontal x_1-axis, and then reflects points through the line $x_1 = x_2$.
   (a) Determine the standard matrix for T.
   (b) Show that T is a rotation about the origin and determine the angle of rotation.

4. Let $\mathbf{u} = \begin{bmatrix} -2 \\ 3 \\ -4 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$.
   (a) Compute $\mathbf{u}^T\mathbf{v}$ and $\mathbf{v}^T\mathbf{u}$.
   (b) Compute $\mathbf{u}\mathbf{v}^T$ and $\mathbf{v}\mathbf{u}^T$.

5. Let $A = \begin{bmatrix} 2 & -3 \\ 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ -5 & k \end{bmatrix}$. Determine the value(s) of k, if any, for which $A\,B = B\,A$.

6. Let $A = \begin{bmatrix} 3 & -6 \\ -1 & 2 \end{bmatrix}$.
   (a) Construct a 2×2 matrix B with two distinct nonzero columns such that $A\,B = O$, the zero matrix.
   (b) Construct a 2×2 matrix C with two distinct nonzero columns such that $C\,A = O$, the zero matrix.
   (c) Does $A\,C = B\,A$ for the matrices B and C you found above?