Math 18 **Homework 4** **Due February 10, 2020**

1. Let A, B, and X be $n \times n$ matrices such that A, X, and $A - AX$ are invertible with $(A - AX)^{-1} = X^{-1}B$.
   (a) Explain why B is invertible.
   (b) Solve the equation $(A - AX)^{-1} = X^{-1}B$ for X. If you need to invert a matrix, explain why that matrix is invertible.

2. Let A be an invertible matrix. Explain why the columns of A^{-1} are linearly independent.

3. Let W be the set of all vectors of the form $\begin{bmatrix} s+3t \\ s-t \\ 2s-t \\ s+t \end{bmatrix}$.
   (a) Show that W is a subspace of $\mathbb{R}^4$.
   (b) Let $\mathbf{v} = \begin{bmatrix} 9 \\ 1 \\ 4 \\ 5 \end{bmatrix}$. Determine whether or not $\mathbf{v} \in W$.

4. Let

$$A = \begin{bmatrix} 1 & 5 & -4 & -3 & 1 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

   Find a linearly independent set of vectors that span $\mathrm{Nul}(A)$, the null space of A.

5. The following matrices A and B are row equivalent.

$$A = \begin{bmatrix} 1 & 2 & 1 & 11 & -3 \\ 2 & 4 & 1 & 15 & 2 \\ 1 & 2 & 0 & 4 & 5 \\ 3 & 6 & 1 & 19 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 0 & 4 & 0 \\ 0 & 0 & 1 & 7 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

   (a) Find a basis for $\mathrm{Nul}(A)$, the null space of A.
   (b) Find a basis for $\mathrm{Col}(A)$, the column space of A.

6. Let V be an n-dimensional vector space. Suppose $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is a subset of V containing k vectors with $k < n$. Explain why S cannot span V.