1. The matrices

$$A = \begin{bmatrix} 1 & 1 & -3 & 7 & 9 & -9 \\ 1 & 2 & -4 & 10 & 13 & -12 \\ 1 & -1 & -1 & 1 & 1 & -3 \\ 1 & -3 & 1 & -5 & -7 & 3 \\ 1 & -2 & 0 & 0 & -5 & -4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & -2 & 0 & 9 & 2 \\ 0 & 1 & -1 & 0 & 7 & 3 \\ 0 & 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

are row equivalent.

(a) Find a basis for Row(A), the row space of A.

(b) Find a basis for Col(A), the column space of A.

(c) Find a basis for Nul(A), the null space of A.

(d) Determine the dimension of Nul $\left(A^T\right)$, the null space of A^T.

2. Let $A = [\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3\ \mathbf{a}_4]$ be a 4×4 matrix with reduced echelon form $\tilde{A} = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

If $\mathbf{a}_1 = \begin{bmatrix} -3 \\ 5 \\ 2 \\ 1 \end{bmatrix}$ and $\mathbf{a}_2 = \begin{bmatrix} 4 \\ -3 \\ 7 \\ -1 \end{bmatrix}$, determine $\mathbf{a}_3$ and $\mathbf{a}_4$.

3. The set $\mathcal{B} = \left\{1 + t^2,\ t + t^2,\ 1 + 2t + t^2\right\}$ is a basis for $\mathbb{P}_2$, the vector space of polynomials with degree at most 2. Find the $\mathcal{B}$-coordinate vector for $\mathbf{p} = 6 + 3t + t^2$.

4. Find all values of λ for which $\det \begin{vmatrix} 2-\lambda & 4 \\ 3 & 3-\lambda \end{vmatrix} = 0$.

5. Let A be a $n \times n$ matrix. Explain why each of the following statements is true. Be sure to state the appropriate theorem or theorems that apply.

(a) If A is invertible, then $\det\left(A^{-1}\right) = \dfrac{1}{\det(A)}$.

(b) If $\det\left(A^3\right) = 0$, then A is not invertible.

6. Find the volume of the parallelepiped with one vertex at the origin $(0,0,0)$ and adjacent vertices at $(1,3,0)$, $(-2,0,2)$, and $(-1,3,-1)$.