1. Let $A = \begin{bmatrix} 4 & -2 & -2 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.
   (a) Find a basis for the eigenspace of A corresponding to $\lambda = 1$.
   (b) Find a basis for the eigenspace of A corresponding to $\lambda = 2$.
   (c) Find a basis for the eigenspace of A corresponding to $\lambda = 3$.

2. Suppose λ is an eigenvalue of an invertible matrix A. Show that $\dfrac{1}{\lambda}$ is an eigenvalue of A^{-1}.

3. Let A be a $n \times n$ matrix.
   (a) Show that A and A^T have the same eigenvalues.
   (b) Do A and A^T necessarily have the same eigenvectors? Explain.

4. Let $A = \begin{bmatrix} -1 & -3 & 0 \\ 0 & 4 & 0 \\ 1 & 1 & 2 \end{bmatrix}$.
   (a) Find the characteristic polynomial of A.
   (b) Find the eigenvalues of A and a basis for each of the corresponding eigenspaces of A. (Hint: One of the eigenvalues is $\lambda = 2$.)

5. Let $A = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & h & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

   Find the value(s) of h for which the dimension of the eigenspace of A corresponding to $\lambda = 5$ is 2.

6. Let A be a $n \times n$ matrix with n distinct real eigenvalues $\lambda_1, \ldots, \lambda_n$, so that

   $$\det(A - \lambda I) = (\lambda_1 - \lambda)(\lambda_2 - \lambda) \cdots (\lambda_n - \lambda).$$

   Explain why $\det(A)$ is equal to the product of the n eigenvalues of A.